

Gelfand's impact at the beginning of my mathematical life (1940-1944)

E.B. Dynkin

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In 1940-41 academic year I was a 16 years old freshman in the Department of Mechanics and Mathematics (Mehmat) of Moscow University. All freshmen have taken three mandatory mathematical courses: Mathematical Analysis (Calculus), Algebra and Analytic Geometry given, respectively, by Lev Abramovich Tumarkin, Israil' Moiseevich Gelfand and Boris Nikolaevich Delone. In groups of about 20, students also worked in problem solving sessions lead by Docents (Associate Professors). (There were several groups in the division of Mathematics and several in the division of Mechanics.)

At the beginning, the most popular lecturer was Delone. Tumarkin's lectures were well organized and easy to follow even for the least prepared students. I found them rather boring and preferred to read myself Course d'analysé infinitésimale of Ch.-J. de la Vallee-Poussin (translated into Russian). Gelfand was 27 years old and he received his second degree (Doctor of Physics and Mathematics) only a few months earlier. At the beginning his lectures were not especially inspiring because he emphasized mostly the logical structure of mathematical reasoning (like a construction of a converse statement). However, in a few months, the situation changed dramatically when he started to present linear algebra in the spirit of the functional analysis. At that times textbooks on this subject have been written in a coordinates language (including a popular book of Schreier and Sperner that I studied before). I. M. used instead the language of linear operators in a finite-dimensional Hilbert space. I remember my joy when I solved a problem given by him in class: to prove that two commuting symmetric complex matrices have a joint eigen-vector.

Gelfand's pedagogical method included a stick and a carrot. From time to time he embarrassed a student by calling him or her to the floor and by asking a question to demonstrate a poor understanding of the material. Usually he selected for this treatment students from the top of the class. During oral exams (common in the Soviet Union) he assigned to such a student a rather non-trivial problem. He followed

a partial progress in solution for an hour or more (interrupted by examining other students) and then he gave to the student the highest grade even for a partial success.

His ways of attracting promising students were diverse. In May (just a month before the end of the academic year) he invited me and Sasha Kronrod (who was a third year student) home for a dinner and he suggested that we help him to write a textbook on linear algebra based on his course. Naturally, this was forgotten the next month when the Soviet Union was invaded by the Nazi Germany. [The book “Lectures on Linear Algebra” was written later in collaboration with S. V. Fomin and published in 1948.]

My poor vision made me unfit for the draft, so during two academic years I continued my undergraduate study as a student of the Molotov (Perm’) University. I was readmitted to Moscow University in Fall 1943. I was one of the first members of the celebrated Gelfand’s seminar that started in December 1943 or January 1944. At that time Gelfand started to work on infinite-dimensional linear representations of simple Lie groups, and the Killing-Cartan-Weyl theory of such groups and their linear representation was the main subject of the seminar. Selim Grigor’evich Krein and I were requested to review this theory. Selim presented a simplified version of Killing-Cartan results by restricting himself by compact groups. (His work has never been published. Later the same approach was developed independently in several monographs, including the Second Edition of Pontryagin’s “Topological groups”.) My task was to present H. Weyl-van der Waerden’s papers that contained the most contemporary version of the final step in the classification of semisimple Lie algebras. I found them very difficult to read, and I tried to find my own ways. It came to my mind that there is a natural way to select a set of generators for a semisimple Lie algebra by using simple roots (i.e., roots which cannot be represented as a sum of two positive roots). Since the angle between any two simple roots can be equal only to $\pi/2$, $2\pi/3$, $3\pi/4$, $5\pi/6$, a system of simple roots can be represented by a simple diagram. An article was submitted to *Matematicheskii Sbornik* in October 1944. Only a few years later, when recent literature from the West reached Moscow, I discovered that similar diagrams have been used by Coxeter for describing crystallographic groups.

At the beginning only a few mathematicians participated in the seminar. Many were still in the army or in evacuation from Moscow. But the next year the seminar expanded. I. M. started to recruit young freshmen and sophomores. Among them were Ima Shnol (admitted to MGU at 15) and Albert Molchanov. Ima recalls: “I. M. warned us that at the beginning we will understand nothing. And indeed, we started to understand something only after a half a year.” He also recalls a joking promoting them from “mebel” (furniture) to “chleny” (members).

Unfortunately I do not remember who else (besides Selim and me) were the original members. (The custom prevented me from taking any notes when I emigrated from the USSR.)