SYLLABUS FOR WRITTEN QUALIFYING EXAMS

Prepared by the Ad Hoc Committee on the qualifying exam,
and revised by the Graduate Committee in 2014

Algebra

• Vector spaces and linear transformations, matrices, bases, change of bases.
• Groups and actions of groups on sets. Basic concepts, normal subgroups, symmetric groups, free groups, products of groups, the Sylow theorems.
• Rings, including ideals, Hilbert Basis Theorem, principal ideal domains (PIDs) and unique factorization.
• Modules, including the Jordan-Hölder Theorem and exact sequences. The structure of finitely generated modules over a PID; applications to Jordan and rational canonical forms.
• Bilinear and quadratic forms, including inner product spaces, alternating and symmetric forms.

Complex Analysis and Advanced Calculus

• Differential and integral calculus of one and several variables. This includes differentiability in one and several variables, Riemann integral and its fundamental properties in one variable, line and surface integrals, theorems of Green and Stokes and the divergence theorem, Jacobians, implicit and inverse function theorems, and elementary differential equations.
• The complex derivative, holomorphic (i.e., analytic) functions on an open set, Cauchy-Riemann equations, harmonic functions, conformal maps, mapping properties of the exponential and logarithm.
• Complex integration. The Cauchy Integral Theorem and the Cauchy Integral Formula. The residue theorem and evaluation of definite integrals.
• Taylor and Laurent expansions of holomorphic functions. Behavior near isolated singularities.
• The Cauchy inequalities and the maximum principle.
• The Riemann sphere and the extended complex plane, fractional linear transformations (i.e., Möbius mappings). Statement of the Riemann Mapping Theorem.

Real Analysis and Elementary Point Set Topology

• Basic properties of the reals and metric spaces. This includes Cauchy sequences, completeness, sequential compactness and compactness, continuity and uniform continuity.
• Basic topological notions such as connectivity, Hausdorff spaces, compactness, product spaces and quotient topologies.
• Sequences and series of numbers and functions, including absolute and uniform convergence, and equicontinuity.
• Definition and elementary properties of Lebesgue measure.
• Measurable functions, simple functions.
• The Lebesgue integral and its elementary properties.
• Convergence theorems.
• Various types of convergence, such as almost everywhere, in measure, in mean, and in L^p.
• Multiple integrals and changing the order of integration (Tonelli and Fubini Theorems).
• Absolutely continuous functions and functions of bounded variation.