Major Topic: Finite-Dimensional Lie Algebras

- **Elementary notions and basic theory**
  - Definitions, examples, representations, modules
  - Solvable, nilpotent, simple, semisimple Lie algebras and the Killing form
  - Engel’s Theorem and Lie’s Theorem
  - Cartan’s criteria for semisimplicity and solvability
  - Semisimple Lie algebras as direct products of simple Lie algebras
  - Weyl’s Theorem for complete reducibility of modules for semisimple Lie algebras

- **Semisimple Lie algebras**
  - Representations of $\mathfrak{sl}(2, \mathbb{C})$
  - Root systems and axiomatics
  - Simple roots and the Weyl group
  - Classification of root systems

- **Representation theory**
  - Universal enveloping algebras
  - Poincaré-Birkhoff-Witt Theorem
  - Serre’s theorem
  - Construction of all finite-dimensional modules for semisimple Lie algebras
MINOR TOPIC: VERTEX OPERATOR ALGEBRAS

• Definitions and properties
  – Formal calculus
  – Axiomatic definitions and basic properties of vertex (operator) algebras
  – Axiomatic definitions and basic properties of modules for vertex (operator) algebras
  – Analytic definitions of vertex operator algebras and modules (as in [2])
  – Equivalence of Jacobi identity with commutativity and associativity

• Examples of vertex operator algebras
  – Huang’s analytic construction theorem for grading-restricted vertex algebras
  – Vertex operator algebras associated to the Virasoro algebra (construction using [2], results as in [1], Section 6.1)
  – Vertex operator algebras associated to affine Lie algebras (construction using [2], results as in [4], Section 6.2)
  – Vertex operator algebras and modules associated to lattices (as in [1])

References


