Oral Qualifying Exam Syllabus

Xiao Chen

1 Mathematical Finance

a. Risk-Neutral Pricing
Risk-Neutral Measure
Fundamental Theorems of Asset Pricing

b. Connections with PDE
Feynman-Kac Theorem
Black-Scholes-Merton Equation

c. Options
Knock-out barrier option
Lookback option
Asian option
American option, American perpetual put option

d. Jump Process
Poisson process, Compound Poisson process, Jump process
Ito's formula for jump process
Change of measure
Pricing a European call in jump model

e. Black and Scholes Model
Construction of self-financing strategies
Absence of arbitrage
f. Arbitrage-free Pricing in Black and Scholes Model

Black and Scholes PDE for standard European options
Risk-neutral valuation for European options
Valuation of American options

g. Optimal investment in Black and Scholes model

The Hamilton-Jacobi-Bellman equation
The martingale characterization of optimal investment strategy

2 Stochastic Calculus

a. Stochastic Process

Filtration, Stopping time, Optional time
Martingale, Submartingale, Supermartingale, Local martingale
Upcrossing inequality, Submartingale convergence theorem
Doob’s maximal inequality
Optional Sampling Theorem
Doob’s-Meyer Decomposition
Square-integrable Martingales, Quadratic variation

b. Brownian Motion

Brownian Motion, Construction of Brownian Motion
The invariance principle
Markov Property, Reflection Principle
Distribution of Brownian Motion and its running maximum
Distribution of first passage time
Strong law of large number for standard Brownian Motion
The law of iterated logarithm

c. Stochastic Integration

Construction of stochastic integral
Ito’s rule, Martingale characterization of Brownian Motion
The Burkholder-Davis-Gundy inequalities
Girsanov Theorem, Novikov Condition
d. Stochastic Differential Equations

Strong solutions, Existence and uniqueness
Weak solutions, Existence and uniqueness

References

[1] Steven E. Shreve, Stochastic Calculus for Finance II

[2] Ioannis Karatzas, Steven E. Shreve, Brownian Motion and Stochastic Calculus, second edition