Oral Qualifying Exam Syllabus

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Major Topics: Lie Algebras

1. Basic Concepts
   (1) Definitions and Examples
   (2) Solvable, Nilpotent, Simple, and Semisimple Lie Algebras
   (3) Engel’s Theorem

2. Semisimple Lie Algebras and Root Systems
   (1) Theorems of Lie and Cartan
   (2) Killing Form, Semisimplicity and Simplicity
   (3) Weyl’s Theorem
   (4) Representations of $\mathfrak{sl}_2$
   (5) Root Space Decomposition and Root Systems
   (6) Simple Roots and Weyl Groups
   (7) Classification of Irreducible Root Systems

3. Existence Theorem
   (1) Universal Enveloping Algebra and Poincaré–Birkhoff–Witt Theorem
   (2) Serre’s Theorem

4. Representation of Semisimple Lie Algebras in the BGG Category $\mathcal{O}$
   (1) Basic Definitions of the Category $\mathcal{O}$
   (2) Highest Weight Modules, Verma Modules, and Irreducible Finite Dimensional Modules
   (3) Harish-Chandra’s Theorem
   (4) Characters of Finite Dimensional Modules
Minor Topic: Representations of Finite Groups

1. Basic Concepts
   (1) Definitions and Examples
   (2) Irreducible Modules and Schur’s Lemma
   (3) Complete Reducible Modules and Maschke’s Theorem
   (4) Semisimple Algebras and Wedderburn’s Theorem

2. Complex Representations and Characters
   (1) Irreducible Characters
   (2) Orthogonality Relations
   (3) Character Table and Group Structure

3. Products of Characters
   (1) Tensor Products
   (2) Frobenius–Schur Indicator

4. Induction and Restriction
   (1) Frobenius Reciprocity
   (2) Clifford’s Theorem
   (3) Clifford Correspondence and Gallagher’s Theorem

References

