Oral Qualifying Exam Syllabus

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1 Major Topic: Conley Index Theory

- **Attractor-repeller pairs**: Definitions, Attractor-repeller pair decompositions, compatibility with Lyapunov functions.

- **Morse decompositions**: Definitions, compatibility with Lyapunov functions, Conley form, duality.

- **Conley’s decomposition theorem**: Chain recurrence, Conley’s decomposition theorem.

- **Conley index for flows**: Homotopy Conley index, independence of index pair, homology Conley index, continuation property, hyperbolic fixed point case, application of Thom isomorphism theorem.

- **Conley index for maps**: Index map, homotopy Conley index, shift-equivalence for choices of index pairs, homology Conley index, continuation property, hyperbolic fixed point case, compatibility with Morse index.

- **Connection matrices**: Connecting homomorphism, connection matrix for attractor-repeller decompositions, connection matrix for Morse decompositions, existence of connection matrix.

- **Transition matrices**: Basic definition, singular transition matrices, topological transition matrices.

References


2 Minor Topic: K-Theory

- **Basic definitions for vector bundles**: Vector bundles, sections, morphisms.
- **Operations, sub-bundles, quotient bundles**: Direct sum, tensor, dual, Hom, and exterior power bundles. Definitions for sub-bundles, strict homomorphisms, Hermitian bundle.
- **Vector bundles on compact spaces**: Homotopy invariance, trivialization of bundles, existence of metrics on vector bundles, classifying bundle, homotopy theoretic definition for $\text{Vect}_n(X)$.
- **Definitions for K-Theory**: Grothendieck completion definition, homotopy-theoretic definitions for $K(X)$.
- **Elementary properties**: Relative K-theory definitions, exact sequence for relative K-theory, retracts, statement of Bott periodicity.
- **Computations of $K^*(X)$**: $K^*(X)$ for cell complexes, projective bundles, etc.
- **Multiplication in $K^*(X,Y)$**: Definition for multiplication in $K^*(X)$, $K^*(X,Y)$ as a $K(X)$–module, Euler characteristic.
- **The Thom isomorphism**: Thom isomorphism, splitting principle.

References
