ORAL QUALIFYING EXAM SYLLABUS

YUCHEN WEI

• Major Topic: Symmetric Functions
  (1) The ring of symmetric functions $\Lambda$.
    • Elementary symmetric functions $e_r$.
    • Complete symmetric functions $h_r$.
    • Power sums $p_r$.
    How to prove
    \[ \Lambda = \mathbb{C}[e_0, e_1, e_2, \ldots] = \mathbb{C}[h_0, h_1, h_2, \ldots] = \mathbb{C}[p_0, p_1, p_2, \ldots] \]
    How to express $e_r$, $h_r$, $p_r$ in terms of each other.
  (2) Partitions and semistandard tableaux, Robinson-Schensted-Knuth bijection, row and column insertions, plactic monoid, connection with increasing subsequences.
  (3) Schur functions $S_\lambda$. Equivalent definitions
    \[ \sum_T x^T \]
    \[ \det (x^{\lambda} + n - j) / \prod_{1 \leq i < j \leq n} (x_i - x_j) \]
  (4) Complex representations of $S_n$.
  (5) Schur functions from the representation theory of $S_n$ and of $GL_n(\mathbb{C})$, Schur-Weyl duality.
  (6) Cauchy identity, Pieri formulas, Jacobi-Trudi formulas, Hook length formula, Weyl dimension formula.
  (7) The Littlewood-Richardson rule, skew Schur functions.
  (8) Hall-Littlewood symmetric functions, connections to representation theory of $GL_n$ over a finite field and Hecke rings.
  (9) Macdonald symmetric functions. Operators $D'_n$. Connections to double affine Hecke algebras.
  (10) The $(q, t)$-Kostka coefficients $K_{\lambda\mu}(q, t) \in \mathbb{Z}[q, t]$ is a polynomial in $q$ and $t$ with integral coefficients.

• Minor Topic: Lie Algebras
  (1) Basic theory of Lie algebras.
    • Definitions, representations, modules
    • Solvable, nilpotent, semisimple Lie algebras
    • Engel’s theorem
  (2) Semisimple Lie algebras.
    • Lie’s theorem and Cartan’s criterion
• Killing form
• Weyl’s theorem

(3) Root systems.
  • Definition of root systems
  • Simple roots and the Weyl group
  • Classification of root systems

(4) Existence Theorem.
  • Universal enveloping algebras
  • Poincaré-Birkhoff-Witt theorem
  • Serre’s theorem

REFERENCES


