1 Major: Partial Differential Equations

1.1 Laplace’s Equation
(Refer to [2] section 2.8 for sub and super harmonic functions and Perron’s method and [1] section 2.2 for the rest)

• Fundamental solution, Poisson’s equation
• Mean-value formulas
• Properties of harmonic functions: Maximum principles, sub and super harmonic functions, Perron’s method
• Green’s function
• Energy methods

1.2 Heat Equation
(Refer to [1] section 2.3)

• Fundamental solution
• Mean-value formula
• Properties of solutions, maximum principles
• Energy methods

1.3 Sobolev Spaces
(Refer to [1] section 5.1-5.7 and 5.8.1)

• Weak derivatives
• Definition and elementary properties
• Approximation by smooth functions

• Extensions

• Traces

• Sobolev inequalities: Gagliardo-Nirenberg-Sobolev inequality, Morrey’s inequality, General Sobolev inequalities

• Compactness: Rellich-Kondrachov compactness theorem, Arzela-Ascoli compactness criterion

• Poincaré's inequalities

1.4 Second Order Elliptic Equations
(Refer to [1] section 6.1-6.4)

• Definitions of elliptic equations and weak solutions

• Existence: Lax-Milgram theorem, energy estimates, Fredholm alternative

• Regularity

• Maximum principles: Weak maximum principle, Hopf’s lemma, strong maximum principle

• Harnack’s inequality

1.5 Other Definitions and Statements (without Proof)

• Hölder spaces

• Schauder estimates

• The continuity method

• Calderon-Zygmund inequality

• viscosity solution

2 Minor: Riemannian Geometry and Basics of Minimal Surfaces

2.1 Riemannian Geometry
(Based on the differential geometry course taught by Professor Daniel Ketover)

• Affine connection, Levi-Civita connection
• Geodesics, parallel transport, minimizing properties, exponential map, normal coordinates, first variation formula

• Hopf-Rinow theorem

• Sectional curvature, Ricci curvature, scalar curvature

• Jacobi fields, conjugate points

• The second variation formula, the Bonnet-Myers theorem

• Isometric immersions, second fundamental form

2.2 Basics of Minimal Surfaces
(Refer to [6] section 1.1-1.8)

• Minimal surface equation

• First variation formula

• Bernstein’s theorem

• The strong maximum principle

• Second variation formula

References


