Oral Qualifying Exam Syllabus

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1. Symplectic Geometry

1) **Symplectic Manifolds**: Symplectic Vector Spaces, Symplectic Structure on Manifold, Tautological Symplectic form on Cotangent Bundle, Symplectic Volume, Lagrangian Submanifold, Symplectic and Hamiltonian Isotopies

2) **Local Theory of Symplectic Manifolds**: Moser’s Trick, Weinstein Lagrangian Neighborhood, Darboux Coordinates, Isotopic and Strongly Isotopic Symplectic Structures

3) **Almost Complex Structure**: Existence of compatible Almost Complex Structure, Complex manifolds, Kähler forms, basics of Dolbeault Theory.

4) **Hamiltonian Dynamics and Moment Maps**
   - Hamiltonian Vector fields
   - Symplectic and Hamiltonian actions, Moment maps, Orbit spaces, MarsdenWeinstein Reduction, Convexity of Moment map, Symplectic Structure on the Space of Connections, Action of Gauge group (Example of Circle Bundle), Existence and Uniqueness of Moment Maps

References:
1) Lectures on Symplectic Geometry, Ana Cannas da Silva
2) Introduction to Symplectic Topology, McDuff and Salamon

2. Riemannian Geometry

(1) Riemannian metric
(2) Levi-civita connection
(3) Curvature tensor, sectional curvature, ricci curvature, scalar curvature
(4) Geodesics and exponential map
(5) Jacobi fields, conjugate points
(6) First variation and second variation formulas and consequences,
(7) Bonnet-Myers Theorem, Hopf-Rinow Theorem, Cartan-Hadamard Theorem, Synge Theorem,

Reference:
1) Riemannian Geometry by Manfredo P do Carmo
2) Introduction to Smooth Manifolds by John M Lee.