ORAL QUALIFYING EXAM SYLLABUS

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1. Schemes

1. Sheaves: Definition, presheaves and associated sheaves, morphisms, stalks, pull-back and push-forward

2. Schemes: Affine schemes, Proj, structure sheaf, scheme associated to a variety

3. First properties of schemes: Reduced, integral, Noetherian schemes, morphisms of finite type, finite morphisms, closed immersions, dimension, fibre products

4. Separated and proper morphisms: Definition of separated and proper morphisms, valuative criteria, projective morphisms, reduced structure of closed subsets, scheme-theoretic image, constructable sets

5. Coherent and quasi-coherent sheaves: Definition of \( \mathcal{O}_X \)-modules, quasi-coherent and coherent sheaves, constructions on \( \mathcal{O}_X \)-modules, invertible sheaves, vector bundles

6. Divisors: Weil divisors, Cartier divisors, equivalence for locally factorial schemes, invertible sheaves

7. Projective morphisms: Criteria and characterization of projective morphisms, ample and very ample line bundles, blowups

8. Differentials: Derivations, module of relative differential forms, sheaves of differentials and nonsingularity, tangent sheaf, canonical sheaf, geometric genus

2. Cohomology

1. Derived functors: Abelian categories, complexes, derived functors, \( \delta \)-functors

2. Cohomology of sheaves: The category of sheaves of \( \mathcal{O}_X \)-modules has enough injectives, some basic vanishing theorems

3. Cohomology of a Noetherian Affine Scheme: vanishing of higher cohomology of quasi-coherent sheaves and Serre’s criterion for being affine in terms of vanishing of cohomology

4. Cech Cohomology: Definition, isomorphism with regular cohomology for a noetherian separated scheme

5. Cohomology of projective space: Calculation using Cech cohomology, Serre vanishing, cohomological criterion of ampleness

6. Ext Groups and sheaves: Definition of basic properties
7. **Serre duality**: dualizing sheaf and duality for a projective scheme, invertibility of dualizing sheaf for local complete intersection, isomorphism between dualizing sheaf and canonical sheaf for nonsingular projective variety.

8. **Spectral Sequence**: Filtered complex, double complex

9. **Hypercohomology**

### 3. Complex Algebraic Surfaces

1. **Cohomology**: Riemann-Roch Theorem, Noether's formula, and the genus formula

2. **Del Pezzo Surfaces**: Definitions and blow-ups

3. **K3 surfaces**: Definition, cohomology, periods, polarizations, examples

4. **Enriques surfaces**: Examples and correspondence with K3 surfaces

5. **ADE singularities**: Quotients, equations and resolutions

6. **Enriques-Kodaira classification**

7. **Bogomolov-Miyaoka-Yau inequality**

### References


