Oral Qualifying Exam Syllabus

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1 Major topic: Affine Lie algebras and representations

1.1 Constructions and realizations

- Free Lie algebra construction from positive semi-definite generalized Cartan matrix
- Loop realizations of twisted and untwisted affine Lie algebras
- Equivalence of the two constructions

1.2 Modules

- Definition of standard modules
- Weyl-Kac character formula for affine Lie algebras
- Denominator formula, Macdonald's identities and the Jacobi triple product identity
- Numerator formula and the principally specialized character formula
- Examples of the q-series

1.3 Vertex operator realizations

- Formal calculus
- Construction of the two level-1 standard modules for $\widehat{\mathfrak{sl}(2)}$ (twisted and untwisted realizations)
- Equivalence of two constructions
- Construction of standard level-1 modules for simply-laced untwisted affine Lie algebras (untwisted realization)

2 Minor topic: Semisimple Lie algebras

2.1 Basic theory of Lie algebras

- Definitions, examples, representations
- Lie's theorem and Engel's theorem
- Cartan's criterion for semisimplicity
- Universal enveloping algebra and Poincaré-Birkhoff-Witt Theorem

2.2 Semisimple Lie algebras

- Representations of $\mathfrak{sl}(2, \mathbb{C})$
- Cartan decomposition of semisimple Lie algebras
- Root systems and axiomatics
- Serre's theorem
- Realizaton of A, D, E Lie algebras via lattices
- Realization of B, C, F, G Lie algebras as fixed points of A, D, E Lie algebras

2.3 Representation theory of Lie algebras

- Weyl's complete reducibility theorem
- Classification of finite-dimensional irreducible modules
- Weyl's character formula

References

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[FLM] I. Frenkel, J. Lepowsky, A. Meurman, Vertex Operator Algebras and the Monster, Academic Press, 1988

[H] J. E. Humphreys, Introduction to Lie Algebras and Representation Theory, Springer, 1972

[L] J. Lepowsky, Lectures on Kac-Moody Lie Algebras, Université Paris, 1978