Oral qualifying exam syllabus

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1 Primary topic: Combinatorics

1.1 Combinatorics

• **Enumeration**: Bijective, binomial and multinomial coefficients, inclusion-exclusion, Bonferroni inequalities, Stirling’s approximation, Prüfer codes, generating functionology

• **Set systems**: Sperner’s theorem, Fisher’s inequality, Uniform and non-uniform Ray-Chaudhuri–Wilson, Frankl-Wilson, Disproof of Borsuk’s conjecture, Erdős-Ko-Rado, Kruskal-Katona, vertex and edge isoperimetry in the cube, VC dimension, Baranyai’s theorem

• **Posets**: Dilworth, distributive lattices, Birkhoff representation theorem

• **Ramsey theory** Ramsey’s theorem, infinite Ramsey, probabilistic lower bounds, Hales-Jewett, van der Waerden, Erdős-Szekeres

1.2 Graph Theory

• **Connectivity**: Menger’s theorem, max-flow-min-cut

• **Matchings**: Hall’s theorem, König, Tutte’s one-factor theorem, Birkhoff-von Neumann

• **Planarity**: Euler’s formula, Kuratowski’s theorem, the crossing lemma, five-color theorem

• **Coloring**: Existence of graphs with high girth and high chromatic number, Brooks’ theorem, Vizing’s theorem, Thomassen’s five-list-coloring of planar graphs, Galvin’s proof of the Dinitz conjecture, Lovász’s proof of the weak perfect graph theorem, statement of strong perfect graph theorem

• **Extremal**: Turán’s theorem, statement of Szemerédi’s regularity lemma, Erdős-Stone, counting lemma, triangle removal
1.3 Probabilistic Methods

- **Basics**: Linearity of expectation, law of total probability, alterations, Markov, Chebychev, Chernoff

- **Second Moment Method**: General procedure, application to threshold for containing a fixed subgraph in \( G_{n,p} \)

- **Lovász Local Lemma**: Statement for symmetric and general case, application to Ramsey lower bounds, application to discrepancy

- **Correlation Inequalities**: Harris, Kleitman, Ahlswede-Daykin, FKG

- **Martingales and Tight Concentration**: Azuma’s inequality, vertex and edge exposure, application to chromatic number of \( G_{n,1/2} \), Talagrand’s inequality

- **The Poisson Paradigm**: Brun’s sieve, the Janson inequalities, application to the number of triangles or isolated vertices in \( G_{n,p} \)

- **Entropy**: Basic properties, Shearer’s lemma, application to Minc conjecture

2 Convex and discrete geometry

- **Basics**: Jensen’s inequality, support properties, separation

- **Combinatorial geometry**: Helly’s theorem, Radon’s theorem, Caratheodory’s theorem, the Szemerédi-Trotter theorem

- **Steiner Symmetrization**: The sphericity theorem, monotonicity of key properties, application to isoperimetric, isodiametric, Blaschke-Santaló and Brunn-Minkowski inequalities

- **Brunn-Minkowski Theory**: Prékopa-Leindler inequality, extension of Brunn-Minkowski inequality to non-convex bodies, concentration of measure in Gaussian space, statement of concentration of measure on the sphere, Dvoretzky’s theorem

- **Miscellaneous** John’s ellipsoid theorem