Schemes

1. **Sheaves**: Presheaves and associated sheaves, morphisms of sheaves, stalks, pull-back and push-forward

2. **Schemes**: Spec and Proj, morphisms of schemes, generic points, varieties

3. **First properties of schemes**: Reduced, integral, noetherian, normal schemes, morphisms of finite type, finite morphisms, open and closed immersions, fibre products

4. **Separated and proper morphisms**: Definition of separated and proper morphisms, valuative criteria, scheme-theoretic images

5. **Sheaves of Modules**: Definition of $\mathcal{O}_X$-modules, quasi-coherent and coherent sheaves, constructions of $\mathcal{O}_X$-modules, invertible sheaves, vector bundles, twisted sheaves

6. **Divisors**: Weil divisors, Cartier divisors, Weil and Cartier divisor class groups, equivalence for locally factorial schemes, Picard groups

7. **Projective morphisms**: Definition, criteria and characterization of projective morphisms, ample and very ample invertible sheaves, linear systems, blowing ups

8. **Differentials**: Derivations, sheaves of relative differentials, connection to nonsingularity, (co)normal sheaves, canonical sheaves, geometric genus

Cohomology Theory of Sheaves

1. ** Derived functors**: Abelian categories, complexes, derived functors, $\delta$-functors

2. **Cohomology of sheaves**: The category of sheaves of $\mathcal{O}_X$-modules has enough injectives, Grothendieck’s vanishing theorem
3. **Cohomology of Noetherian Affine Schemes**: Cohomological characterization of noetherian affine schemes

4. **Čech Cohomology**: Definition, computation of sheaf cohomology groups for a noetherian separated scheme

5. **Cohomology of Projective Spaces**: Properties of cohomology groups of projective schemes, criterion for ampleness

6. **Extension Groups and Sheaves**

7. **Serre Duality Theorem**: Duality for $\mathbb{P}^n_K$, dualizing sheaves, duality for projective schemes

---

**Curves**

1. **Nonsingular Curves**

2. **Divisors on Curves**

3. **Riemann-Roch**

4. **Hurwitz’ Theorem**

5. **Embeddings in Projective Space**

6. **Elliptic Curves**: The j-invariant, the group structure, the Jacobian variety, Elliptic functions, the Hasse invariant, rational points on an elliptic curve

---

**The Geometry of Flag Varieties (of Type A)**

1. **Grassmannians and the Plucker Embedding**

2. **Flag Varieties, Bruhat Decomposition and Schubert Varieties**

3. **Schubert Classes, Kleiman’s Transversality, and Intersections**

4. **Line Bundles and Characters, Borel-Weil**

5. **Singularities of Schubert Varieties**: Normality, rationality of singularities, cohomology of line bundles

6. **The Grothendieck Ring and a Degeneration of the Diagonal of a Flag Variety**

7. **Positivity of the Structure Constants of the Grothendieck Ring**

---

**Linear Algebraic Groups (over Algebraically Closed Fields)**
1. **Definitions and First Properties**: Definition of algebraic groups and functor of points, morphisms, G-spaces, Jordan Decomposition, Representations

2. **Commutative Algebraic Groups**

3. **Lie Algebras**

4. **Homogeneous Spaces and Quotients**

5. **Parabolic Subgroups, Borel Subgroups, Solvable Groups**

6. **Classification of Reductive Groups**

**References**


