1 Major Topic: Mean Curvature Flow

The Basics

- First variation of area functional
- Definitions of the flow: Standard definition, invariance under tangential perturbation, definition for graphs
- Special Solutions: shrinkers, expanders, translating solutions
- Proof of short time existence

The Maximum Principle and its Consequences

- The Maximum Principle (for Mean Curvature Flow)
- The Comparison Principle
- The evolution of geometric quantities under mean curvature flow
- Consequences of the evolution equations
- Preservation of: Convexity, mean convexity, k-convexity, etc

Huisken’s Monotonicity Formula and its Consequences, Type 1 Singularities

- Huisken’s Monotonicity Formula
- Rescaling around type 1 singularities
- Analysis of Singularities
- Singularity Models for Mean Convex Hypersurfaces

Type 2 Singularities

- Hamilton’s Blow-up
• The Mean Convex Case
• Hamilton’s Harnack Estimates for Mean Curvature Flow

Regularity Theory at the First Singular Time
• Lower Bound on Area Ratio / Clearing Out Lemma
• White’s Gap Theorem
• Brakke’s Regularity Theorem

2 Minor Topic: Partial Differential Equations

• Laplace’s Equation
  – Representation formula for the solution of the dirichlet problem on the ball
  – Mean Value Property for subharmonic functions
  – Other Properties of Harmonic Functions: Strong Maximum Principle, regularity, Harnack’s inequality, Green’s function

• The Heat Equation
  – Maximum Principles
  – Fundamental Solution of the Heat Equation
  – Other properties of solutions: Strong maximum principle, uniqueness of solution

• Second-Order Elliptic Equations
  – Hopf’s Maximum Principle
  – Alexandrov and Bakelman’s Maximum Principle
  – Maximum Principles for Nonlinear PDE

3 References

[E] Klaus Ecker, Regularity Theory for Mean Curvature Flow
[J] Jurgen Jost, Partial Differential Equations (Second Edition) Ch 1, 2, 4
[M] Carlo Mantegazza, Lecture Notes on Mean Curvature Flow