Qualifying Exam Syllabus

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Quadratic Forms over Fields of Characteristic ≠ 2 (Major Topic)

• Foundations
  – Definitions
  – Hyperbolic spaces
  – Witt decomposition theorem and Witt cancellation theorem
  – Chain equivalence
  – Generation of the orthogonal group by reflections

• Witt rings
  – Definition of $\hat{W}(F)$ and $W(F)$
  – Group of square classes
  – Examples of Witt rings

• Quaternion algebras and their norm forms
  – Quaternion algebras as quadratic spaces
  – Coverings of the orthogonal groups
  – Linkage of quaternion algebras and Albert’s theorem

• The Brauer-Wall group
  – Central simple algebras (CSA) and the Brauer group
  – Central simple graded algebras (CSGA)
  – Structure theory of CSGA
  – The Brauer-Wall group

• Local fields and global fields
  – Springer’s theorem for complete discretely valued (c.d.v.) fields
  – Quadratic forms over local fields
  – Hasse-Minkowski principle
  – Witt ring of $\mathbb{Q}$
  – Hilbert reciprocity and quadratic reciprocity
Lie Algebras (Minor Topic)

- Foundations
  - Definitions, examples, representations, and modules
  - Solvable, nilpotent, simple, and semisimple Lie algebras, and the Killing form
  - Engel’s Theorem and Lie’s Theorem
  - Cartan’s criteria for semisimplicity and solvability
  - Semisimple Lie algebras as direct products of simple Lie algebras
  - Weyl’s Theorem for complete reducibility of modules for semisimple Lie algebras

- Semisimple Lie algebras
  - Representations of $\mathfrak{sl}(2, \mathbb{C})$
  - Root systems and axiomatics
  - Simple roots and the Weyl group
  - Classification of root systems

- Representation theory
  - Universal enveloping algebras
  - Poincaré-Birkhoff-Witt Theorem
  - Serre’s theorem
  - Construction of all finite-dimensional modules for semisimple Lie algebras

References

