Qualifying Exam Syllabus

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Quadratic Forms over Fields of Characteristic $\neq 2$ (Major Topic)

- Foundations
  - Definitions
  - Hyperbolic spaces
  - Witt decomposition theorem and Witt cancellation theorem
  - Chain equivalence
  - Generation of the orthogonal group by reflections
- Witt rings
  - Definition of $\widehat{W}(F)$ and $W(F)$
  - Group of square classes
  - Examples of Witt rings
- Quaternion algebras and their norm forms
  - Quaternion algebras as quadratic spaces
  - Coverings of the orthogonal groups
  - Linkage of quaternion algebras and Albert's theorem
- The Brauer-Wall group
  - Central simple algebras (CSA) and the Brauer group
  - Central simple graded algebras (CSGA)
  - Structure theory of CSGA
  - The Brauer-Wall group
- Local fields and global fields
  - Springer's theorem for complete discretely valued (c.d.v.) fields
  - Quadratic forms over local fields
  - Hasse-Minkowski principle
  - Witt ring of $\mathbb{Q}$
  - Hilbert reciprocity and quadratic reciprocity

Lie Algebras (Minor Topic)

- Foundations
  - Definitions, examples, representations, and modules
  - Solvable, nilpotent, simple, and semisimple Lie algebras, and the Killing form
  - Engel's Theorem and Lie's Theorem
  - Cartan's criteria for semisimplicity and solvability
  - Semisimple Lie algebras as direct products of simple Lie algebras
  - Weyl's Theorem for complete reducibility of modules for semisimple Lie algebras
- Semisimple Lie algebras
  - Representations of $\mathfrak{sl}(2, \mathbb{C})$
  - Root systems and axiomatics
  - Simple roots and the Weyl group
  - Classification of root systems
- Representation theory
  - Universal enveloping algebras
  - Poincaré-Birkhoff-Witt Theorem
  - Serre's theorem
  - Construction of all finite-dimensional modules for semisimple Lie algebras

References

1. R. Carter, *Lie Algebras of Finite and Affine Type*, Cambridge University Press, Cambridge, 2005.

2. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer-Verlag, New York, 1972.

3. T. Y. Lam, *Introduction to Quadratic Forms over Fields.* American Mathematical Society, Providence, 2005.