

Oral Qualifying Exam Syllabus

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Analytic number theory

1. The functions $\zeta(s)$ and $L(s, \chi)$
 - analytic continuation, functional equation
 - Hadamard product, explicit formula
 - zero-free region, prime number theorem
 - asymptotic formulas for $N(T)$ and $N(T, \chi)$
 - approximate functional equation, convexity bound
2. Primes in arithmetic progression
 - Siegel-Walfisz theorem
 - Bombieri-Vinogradov theorem
3. Sieve methods
 - Λ^2 sieve, Brun-Titchmarsh estimate
4. Bilinear techniques
 - statements of the additive and multiplicative large sieve inequalities
5. Dirichlet polynomials
 - mean value estimates
 - large values, zero-density estimates for $\zeta(s)$

Algebraic number theory

1. Number fields, rings of integers, integral bases, discriminant
2. Decompositions of primes
3. Minkowski's bound, the class group, the unit theorem
4. Dedekind zeta function, class number formula

References

1. H. Davenport, *Multiplicative Number Theory*, Springer, 1980.
2. H. Iwaniec and E. Kowalski, *Analytic Number Theory*, AMS, 2004.
3. D. Marcus, *Number Fields*, Springer-Verlag, 1977.