The following are review exercises for the Math 111 second exam. These exercises are provided for you to practice or test yourself for readiness for the second exam.

1. Express the following as a simple fraction reduced to lowest terms.
   (a) \( \frac{6}{x-4} + \frac{2}{x-4} \)
   (b) \( \frac{2ab^2 - \frac{3}{ab^3} + \frac{1}{a^2b}}{a^2b + ab - \frac{1}{ab}} \)
   (c) \( \frac{1 - \frac{y}{y-1}}{y + 1 - 1} \)
   (d) \( \frac{\frac{4}{z} - \frac{7}{z}}{\frac{5}{z} - \frac{6}{z - 6}} \)
   (e) \( \frac{\frac{4}{x^2} - \frac{3}{x}}{\frac{1}{x^2} + \frac{3}{3x}} \)

2. Perform the indicated operations and simplify. Express your answers with positive exponents only.
   (a) \( \frac{3y^{-1} + 4x^{-1}}{9y^{-2} - 16x^{-2}} \)
   (b) \( \frac{x^{-1} + y^{-1}}{(x + y)^{-1}} \)
   (c) \( \frac{x^{-3}y - 2x^2y^{-1}}{(xy)^{-1}} \)
   (d) \( (3 - a^{-2})^{-2} \)
   (e) \( (a^{-1/2} + 2b^{1/2})^2 \)

3. Simplify the expression:
   (a) \( \frac{2x(x + 6)^4 - 4x^2(x + 6)^3}{(x + 6)^8} \)
   (b) \( \frac{(1 - x^2)^{1/2} + x^2(1 - x^2)^{-1/2}}{1 - x^2} \)

4. Rationalize the denominator and express your answer in simplest form:
   \( \frac{2(x - y)}{\sqrt{x} + \sqrt{y}} \)

5. Rationalize the numerator and express your answer in simplest form:
   \( \frac{\sqrt{x + h} - \sqrt{x}}{h} \)

6. Solve the following equations for real solution.
   (a) \( \frac{3}{4}x + \frac{5}{6} = 2x - \frac{7}{3} \)
   (b) \( y(y + 2) = 15 \)
   (c) \( 2x^2 = 32 \)
   (d) \( 1 - 2x = 3x^2 \)
   (e) \( 9a^2 + 4 = 12a^2 \)
   (f) \( (x + 12)^2 = 12 \)
   (g) \( (2x + 3)(x - 4) = 3x \)
   (h) \( |2x - 3| + 5 = 7 \)
   (i) \( \frac{1}{3}|5 - x| - 4 = 2 \)
   (j) \( |3x + 4| + 8 = 4 \)
   (k) \( \frac{x}{x + 3} + \frac{5}{x - 7} = \frac{30}{x^2 - 4x - 21} \)
   (l) \( \frac{2}{x - 1} + \frac{3}{4} = \frac{5}{x - 1} \)
   (m) \( \frac{3}{4x^2} + \frac{7}{4x} = \frac{3}{2} \)
   (n) \( \sqrt{5 - x} - 3 = 0 \)
(o) \[ 4 + \sqrt[3]{x-6} = 2 \]  
(p) \[ (x - 2)^{2/3} = 4 \]  
(q) \[ \sqrt{s^2 - 1} = \sqrt{5s - 5} \]

(r) \[ \sqrt{7x + 4} - 2 = x \]  
(s) \[ x^4 - 7x^2 + 12 = 0 \]  
(t) \[ x^{2/3} - 2x^{1/3} - 15 = 0 \]

(u) \[ y^{-6} - 9y^{-3} + 8 = 0 \]  
(v) \[ x + 5\sqrt{x} - 14 = 0 \]  
(w) \[ 3x^{2/3} - 5x^{1/3} - 2 = 0 \]

7. Solve the following for the indicated variable.

(a) \[ \frac{5}{x} = \frac{2}{a} + \frac{3}{b}; \text{ for } x \]  
(b) \[ 3x + y = 4(x - 3y) + 7; \text{ for } x \]

(c) \[ s(r - 3) = 2 - 5(r + t); \text{ for } r \]  
(d) \[ \frac{2w + 3}{3w - 2} = u; \text{ for } w \]

8. Solve the following inequality. Must show complete analysis where applicable. Express the solution using interval notation.

(a) \[ x - 8 > 5x + 3 \]  
(b) \[ 5 \geq \frac{6 - 5x}{3} > 2 \]  
(c) \[ \frac{x - 3}{2} - \frac{x - 1}{4} \geq 1 \]

(d) \[ x^2 - 3x > 18 \]  
(e) \[ 5a^2 \leq 7 - 34a \]  
(f) \[ 2x^2 \geq 11x - 12 \]

(g) \[ \frac{y + 4}{y - 1} < 2 \]  
(h) \[ \frac{3x + 4}{2x + 1} \leq 2 \]  
(i) \[ \frac{2}{x + 1} + \frac{3}{x - 2} \geq \frac{5}{x - 2} \]

(j) \[ |2x + 5| + 1 < 6 \]  
(k) \[ 4|3x - 7| \geq 8 \]  
(l) \[ \frac{-1}{3}|6 - 5x| + 2 \geq 1 \]

9. Find the center and radius of the circle.

(a) \[ (x + 2)^2 + y^2 = 18 \]  
(b) \[ x^2 + y^2 - 2y = 6 \]  
(c) \[ x^2 + y^2 + 4x - 6y - 11 = 0 \]

10. Find the equation of the circle with center \((-3, 2)\) and passing through the point \((4, 5)\)

11. Find the equation of the circle with the end points of the diameter as \((1, 5)\) and \((-4, 3)\).

12. Find the equation of the circle with center \((-1, -3)\) and tangent to the \(x\) axis.

13. The width of a rectangular garden is 8 feet less than its length. If the garden is fenced in with 72 feet of fencing, what are the dimensions of the garden?

14. 1000 tickets were sold. Adult tickets cost $8.50, children’s cost $4.50, and a total of $7300 was collected. How many tickets of each kind were sold?

15. Mrs. B. invested $30,000; part at 5% and part at 8%. The total annual interest from both the
investments was

$2,100. How much did she invest at each rate?

16. How many ounces of a 35% solution of sulfuric acid (and distilled water) must be mixed with 12 oz of a 20% solution to get a 30% solution of sulfuric acid?

17. Two planes leave at 9 AM from airports that are 2700 miles apart and fly towards each other at a speed of 200 mph and 250 mph. At what time will they pass each other?

18. Michael drove to a friend's house at a rate of 45 mph. He came back by the same route, but at a rate of 30 mph. If the round-trip took 5 hours, what is the distance Michael traveled to visit his friend?

19. John traveled north from Miami at an average speed of 50 mph. Two hours after he left, Sharon left from Miami traveling in the same direction at an average speed of 55 mph. How long will it take Sharon to catch up with John?

20. Tom and Jerry have to stuff and mail 1000 envelopes for a new marketing campaign. Jerry can do the job alone in 6 hours. If Tom helps, they can get the job done in 4 hours. How long would it take Tom to do the job by himself?

21. One roofer can put a new roof on a house three times faster than another. Working together they can roof a house in 5 days. How long would it take the faster roofer working alone?

22. A farmer plans to use 180 feet of fencing to enclose a rectangular region, using part of a straight river bank instead of fencing as one side of the rectangle as shown in the figure below. Find the area of the region if the length of the side parallel to the river bank is twice the length of an adjacent side.
ANSWERS

1a) \( \frac{x - 1}{x - 6} \)
1b) \( \frac{b^2 + 2ab - 3a}{4b^2 + a^3b^4 - ab^2} \)
1c) \( \frac{y + 1}{y - 1} \)
1d) \( -\frac{z - 14}{4z - 10} \)
1e) \( \frac{12 - 9x}{3 + 2x} \)

2a) \( \frac{xy}{3x - 4y} \)
2b) \( \frac{(x + y)^2}{xy} \)
2c) \( \frac{y^2 - 2x^5}{x^2} \)
2d) \( \frac{a^4}{9a^4 - 6a^2 + 1} \)
2e) \( \frac{1 + 4a^{1/2}b^{1/2} + 4ab}{a} \)

3a) \( \frac{2x(6 - x)}{(x + 6)^5} \)
3b) \( \frac{1}{(1 - x^2)^{3/2}} \)

4) \( 2(\sqrt{x} - \sqrt{y}) \)
5) \( \frac{1}{\sqrt{x + h} + \sqrt{x}} \)

6a) \( \frac{38}{15} \)
6b) \( y = -5, 3 \)
6c) \( x = \pm 4 \)
6d) \( x = -1, 1/3 \)
6e) \( a = \pm \frac{2\sqrt{3}}{3} \)
6f) \( x = 12 \pm 2\sqrt{3} \)
6g) \( x = 2 \pm \sqrt{10} \)
6h) \( x = 1/2 \) or \( x = 5/2 \)
6i) \( x = 23 \) or \( x = -13 \)
6j) No solution
6k) \( x = 5 \)
6l) \( x = 5 \)

6m) \( z = \frac{-1}{3}, \frac{3}{2} \)
6n) \( x = -4 \)
6o) \( x = -2 \)
6p) \( x = -6, 10 \)
6q) \( s = 4, 1 \)
6r) \( x = 0, 3 \)
6s) \( x = \pm 2, \pm \sqrt{3} \)
6t) \( x = 125, -27 \)
6u) \( y = 1/2, 1 \)
6v) \( x = 4 \)
6w) \( x = -1/27, 8 \)

7a) \( x = \frac{5ab}{3a + 2b} \)
7b) \( x = 13y - 7 \)
7c) \( r = \frac{3s - 5t + 2}{s + 5} \)
7d) \( w = \frac{2u + 3}{3u - 2} \)

8a) \( (-\infty, -11/4) \)
8b) \( [-9/5, 0) \)
8c) \( [9, \infty) \)
8d) \( (\infty, -3) \) U \( (6, \infty) \)
8e) \( [-7, 1/5] \)
8f) \( (-\infty, 3/2) \) U \( [4, \infty) \)
8g) \( (-\infty, 1) \) U \( (6, \infty) \)
8h) \( (-\infty, -1/2) \) U \( [2, \infty) \)
8i) \( (-1, 2) \)
8j) \( (-5, 0) \)
8k) \( (-\infty, 5/3) \) U \( [3, \infty) \)
8l) \( [3/5, 9/5] \)

9a) Center \( (-2, 0) \), radius = \( 3\sqrt{2} \)
9b) Center \( (0, 1) \), radius = \( \sqrt{7} \)
9c) Center \((-2, 3),\) radius \(= 2\sqrt{6}\)

10) \((x + 3)^2 + (y - 2)^2 = 58\)

11) \((x + \frac{3}{2})^2 + (y - 4)^2 = \frac{29}{4}\)

12) \((x + 1)^2 + (y + 3)^2 = 9\)

13) length = 22 ft, width = 14 ft

14) Adult 700, Children 300

15) \$10,000\ at 5\%, \$20,000\ at 8\%

16) 24 oz.

17) At 3:00 PM

18) 90 miles

19) 20 hrs

20) 12 hrs

21) \(6\frac{2}{3}\) days

22) 4050 square feet.