MATH 115 FINAL EXAM REVIEW

SPRING 2019

The problems below summarize many of the essential skills to be successful on your Math 115 final exam. At the same time, the problems are not exhaustive. Other materials may be helpful in assessing your knowledge.

A strong set of algebra skills is a must to best ensure success. Among these are simplifying, adding, subtracting, multiplying, and dividing algebraic expressions related to exponents, radicals, polynomials, and fractions. Factoring and solving equations/inequalities are two other required basic skills.

These skills may be applied to a diverse list of functions. These include linear, quadratic, power, radical, absolute value, reciprocal, polynomial, rational, exponential, logarithmic, and trigonometric.

A. Graphing

Most graphing will be without a calculator. Although at times a graphing calculator may be either helpful or essential to solving a problem.

1. For each problem below determine ***EXACTLY*** (as applicable) the x-intercept(s), y-intercept, vertex, vertical asymptote(s), horizontal asymptote, end behavior, domain and range. Then sketch the graph.

   a. $f(x) = -x^2 + 2x - 3$

   b. $f(x) = x^3 - 4x$

   c. $f(x) = \dfrac{x-2}{x^2 - 2x - 8}$

   d. $f(x) = 3^{x+1} - 1$

   e. $f(x) = \log_3(x+1) + 2$

   Now for 1a., 1c., 1d. and 1e. determine the **EXACT** interval, if any, over which each function is increasing, ***expressing your answer in interval notation.***

   For the next problem sketch the graph and then determine the same applicable attributes as above ***accurate to two (2) decimal places.***

   f. $f(x) = 0.2x^3 - x^2 + 5$

   Now find the interval over which this function is increasing.

2. For each trigonometric function below determine its amplitude, period, phase shift vertical asymptote and range (as applicable). Then sketch one cycle of the graph. Label the maximum/minimum, end points and any other x-intercept(s).

a. $f(x) = 3\sin\left(2x + \frac{\pi}{3}\right)$

b. $f(x) = -2\cos\left(3x - \frac{\pi}{2}\right)$

c. $f(x) = 3\tan\left(2x - \frac{\pi}{2}\right)$

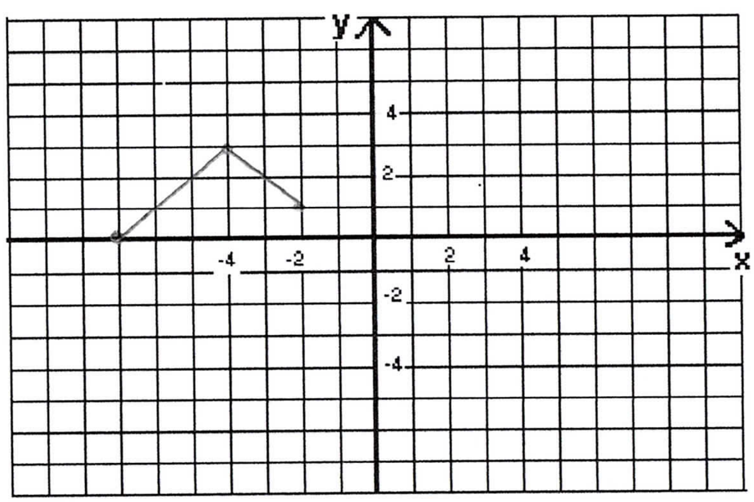
3. For each problem below sketch the graph of the piece-wise defined function. Be sure to label the end points of each piece.

a. $f(x) = \begin{cases} 2 + x & if \ \ x \le -1 \\ 2^{x-3} & if \ \ 3 < x \le 5 \end{cases}$

b. $f(x) = \begin{cases} \sqrt{1-x} & if \ \ -8 < x \le 0 \\ 4 - x^2 & if \ \ 0 < x \le 3 \end{cases}$

4. Based on the graph of the function f below, on the same set of axis, graph

a. $g(x) = -2f(x+1)$ and b. $h(x) = 2f(-x)+1$



B. Evaluating Functions, Domains & Inverses

I. Let $f(x) = \sqrt{x^2 + x - 6}$, $g(x) = -2\log_3(x+1)$, $w(x) = \tan(x)$, and *h(x)* defined by the table

X	h(x)
-2	-3
0	1
3	-2
4	0

1. Determine each of the following ***EXACTLY*** *and in simplest form*:

a. $w\left(\frac{5\pi}{3}\right)$ b. $g(\sqrt[6]{9} - 1)$ c. $(f+g)(2)$ d. $\left(\frac{f}{h}\right)(4)$

e. $(h \circ g)(2)$ f. $g(3^x - 1)$

2. Determine the domain for each of the following, *expressing your answer in interval notation.*

a. *f(x)* b. $g(x)$ c. $\left(\frac{g}{f}\right)(x)$

3. For *f(x)*, *g(x)*, and *h(x)*, respectively, determine whether each is a 1-1 function. And if so find its inverse.

II. Determine ***EXACTLY*** $\tan^{-1}(-\sqrt{3})$.

III. If $\sin t = -\frac{6}{7}$, $270° < t < 360°$, find **EXACTLY**:

1. $\cos t$ 2. $\csc^2 t$ 3. $\sin 2t$ 4. $\sin(t + 30°)$

IV. Find **EXACTLY** $\sin\left(\cos^{-1}\left(\frac{2}{3}\right)\right)$.

V. Evaluate each of the following **EXACTLY**:

1. $\log_{\frac{1}{2}} 16$ 2.. $\log_a \sqrt[3]{7}$, where $\log_a 7 = 0.6$ 3. $\log_3 216 - \frac{1}{2}\log_3 64$

C. Difference Quotients

Let $f(x) = -2x^2 - 3x + 6$ and $g(x) = \dfrac{x}{x-3}$.

Determine each of the following *expressing your answer in simplest form*:

1. $\dfrac{f(x+h) - f(x)}{h}, h \neq 0$

2. $\dfrac{g(x+h) - g(x)}{h}, h \neq 0$

D. Solving Equations

Solve each of the following for the **EXACT** value(s) of x.

1. $3 - 2|x + 5| = 1$

2. $8^{3x} = 16^{x+1}$

3. $5^{2x} = 7^{x+3}$

4. $\log_3(x+2) + \log_3(x-2) = 2$

5. $2\ln e^{|x-1|} - 5 = 7$

6. $2\sin^2 x - \sin x = 0 \quad 0 \leq x < 2\pi$

E. Solving Inequalities

I. Solve each of the following for the **EXACT** values of x, ***expressing your answer in interval notation.***

1. $2 + 3|x - 1| < 5$

2. $3 + 2|x + 1| \geq 7$

3. $x^2 - 2x \geq 8$

4. $(x-3)(x+2)(x-1) < 0$

5. $\dfrac{x+2}{x-1} \leq 0$

II. Which is larger $\sin\left(-\dfrac{\pi}{5}\right)$ or $\cos\left(-\dfrac{\pi}{5}\right)$? Explain your answer.

F. Modeling (Applications)

1. Flying too fast or too slow wastes fuel. The optimum usage, in gallons, can be modeled by the function $F(x) = 0.18x^2 - 1.62x + 780$, where x is the air speed in hundreds of miles per hour, and $F(x)$ is the number of gallons of fuel used at speed x.

    a. ***To the nearest mile per hour***, what air speed will use the minimum amount of fuel?

    b. ***To the nearest gallon***, what is the minimum amount of fuel used?

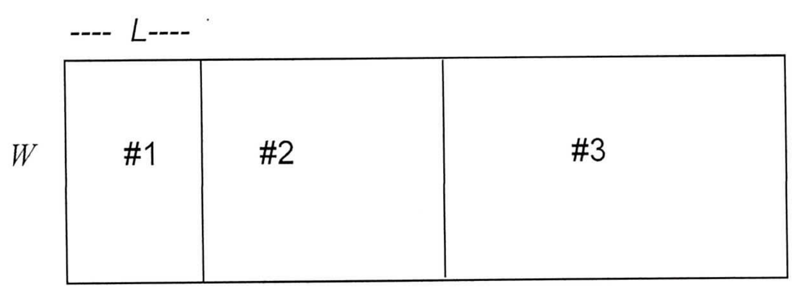
    c. What air speeds(s) would result in the plane using 780 gallons of fuel? ***(Accurate to the nearest mile per hour.)***

2. You have a choice of investing $10,000 in only one of these investments:

    - A 33-month CD paying 4.9% per year compounded quarterly, or
    - A 3-year bond paying 4.7% per year compounded continuously.

    At maturity, which investment will be worth the most? And by how much more? ***(Round to the nearest dollar.)***

3. The diagram below shows three fields. Each individual field has the same width (W). The length of the second field is twice the length of the first field (L), and the length of the third field is triple the length of the second field.



    a. Write an equation for the total area (A) of **the three fields combined** in terms of L and W.

    b. The owner of the fields has 1000 linear feet of wire to fence in the fields. Rewrite the equation for the total area of the three fields combined in terms of the width, W, only.

    c. What is the domain (i.e., possible values of the width) for the total area combined? (*Express your answer in interval notation.*)

    d. Determine the maximum total area for the three fields combined.

4. The initial weight of a mass decays exponentially from 15 grams at the end of 2001 to 10 grams at the end of 2007.

   a. Determine r, the relative annual decay rate of the mass, ***as a decimal accurate to four places.***

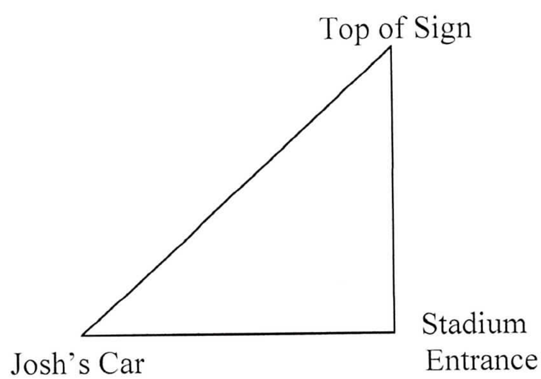
   b. Based on this model, estimate the weight of the mass at the end of 2004, ***accurate to the nearest tenth of a gram.***

   c. If the trend continues, in what calendar year would the weight of the mass be 40 percent of its initial weight?

5. You are standing on top of a lighthouse that is 300 ft high. Looking out at sea with an angle of depression of 14°, you spot a ship in the distance. Determine the distance between yourself at the top of the lighthouse and the ship, ***accurate to one decimal place.***

6. Two boats leave a dock at 1 pm traveling at a 67° angle between their directions of travel. Traveling at speeds of 20 mph and 22 mph, respectively, at 4 pm how far apart are the two boats? ***(Accurate to the nearest mile.)***

7. On football game day, driving on a straight road to High Point Solutions.Com Stadium, Josh looks up at a 37° angle and spots the top of a 200-foot high sign at the stadium entrance.



   a. At the time Josh looks up, how far, ***to the nearest foot***, is he from the entrance to the stadium?

   b. On the same straight road, and at the same time, there is another car 600 feet further away from the stadium entrance that is directly behind Josh's car. From the top of the stadium sign what is the angle of depression to the second car? ***(Accurate to the nearest degree.)*** (Hint: Use results from part a.)

8. In one 12-hour period, the ocean water at a resort, starts at mean sea level, rises to 9 feet above, drops to 9 feet below, and then returns to mean sea level.

   Assuming the same motion repeats itself every 12 hours, find an equation that describes the height *(H)* of the tide above/below mean sea level, in relation to time *(t)*, where time is measured in hours.

9. In 1967, the population of Clements, PA was 7,000, and in 1974 the population was 12,000. Should the population grow linearly, in what calendar year (***to the nearest whole calendar year***) would it reach 50,000? If instead the population grew exponentially, how many fewer years would it take to reach 50,000? ***(Accurate to the nearest integer.)***

G. Miscellaneous

1. Determine the equation of a circle whose center is (-1,3), and whose diameter is $\sqrt{56}$.

2. Determine the **EXACT** value(s) of x for which the coordinate pair $(x, 1)$ is on the circle given by the equation $(x-3)^2 + y^2 = 32$

3. Determine *in simplest form* the slope between coordinates (a, a^2) and $(b, b^2), a \neq b$.

4. Let $f(x) = h^x$ and $g(x) = h^{\sin^2 x}$

   a. Evaluate $f(\tan 2\pi)$.

   b. Solve $h^{\frac{1}{2}} = h^{\cos^2 x}$ **EXACTLY**, for x in the interval $[0, 2\pi)$.

   c. Let $m(x) = \log_h\left(\frac{h}{g(x)}\right)$. Express *m(x)* as a simplified single trigonometric function. {Hint: Consider the properties, identities and laws of logarithms.}

5. Based on a unit circle, if $t = 10$, in what quadrant is the terminal point? What is the value of that terminal point ***(accurate to two decimal places)?***

MATH 115 REVIEWANSWERS

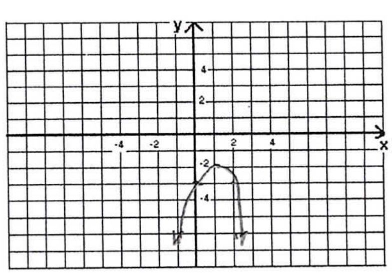
A. Graphing

1.

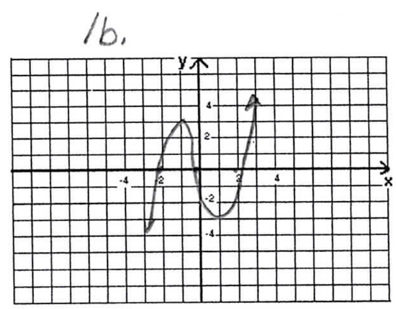
	1a.	1b.	1c.
X- Intercept	None	(-2,0); (0,0); (2,0)	(2,0)
Y-Intercept	$(0,-3)$	(0,0)	(0,1/4)
Vertex	$(1,-2)$	NA	NA
Vertical Asymptote	NA	NA	$x=-2; x=4$
Horizontal Asymptote	NA	NA	$y=0$
End Behavior	As $x \to -\infty, y \to -\infty$; $x \to \infty, y \to -\infty$	As $x \to -\infty, y \to -\infty$; $x \to \infty, y \to \infty$	NA
Domain	$(-\infty,\infty)$	$(-\infty,\infty)$	$(-\infty,-2)\cup(-2,4)$ $\cup(4,\infty)$
Range	$(-\infty,-2]$	$(-\infty,\infty)$	$(-\infty,\infty)$
Increasing	$(-\infty,1]$ or $(-\infty,1)$		Never

	1d.	1e.	1f.
X- Intercept	$(-1,0)$	$(-8/9,0)$	$(-1.90,0)$
Y-Intercept	$(0,2)$	$(0,2)$	$(0,5)$
Vertex	NA	NA	NA
Vertical Asymptote	NA	$x=-1$	NA
Horizontal Asymptote	$y=-1$	NA	NA
End Behavior	NA	NA	As $x \to -\infty, y \to -\infty$; $x \to \infty, y \to \infty$
Domain	$(-\infty,\infty)$	$(-1,\infty)$	$(-\infty,\infty)$
Range	$(-1,\infty)$	$(-\infty,\infty)$	$(-\infty,\infty)$
Increasing	$(-\infty,\infty)$	$(-1,\infty)$	$(-\infty,0)\cup(3.33,\infty)$

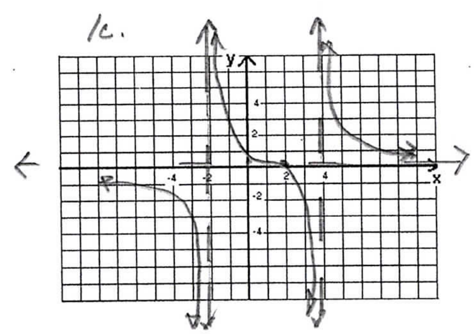
1a.



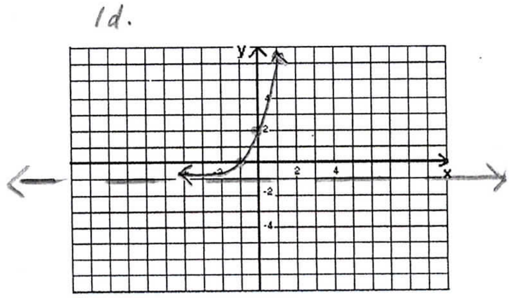
1b.



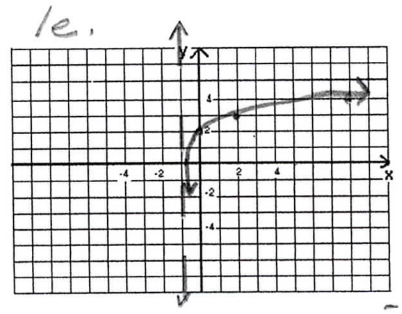
1c.



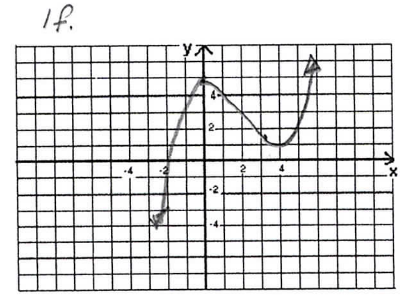
1d.



1e.



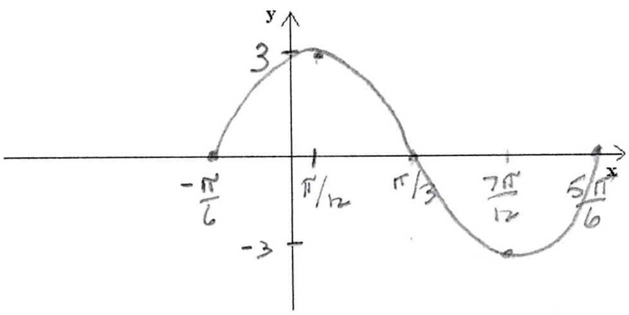
1f.



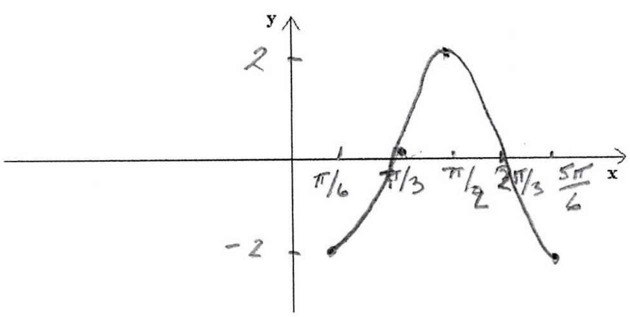
2.

	2a.	2b.	2c.
Amplitude	3	2	NA
Period	π	$\frac{2\pi}{3}$	$\frac{\pi}{2}$
Phase Shift	$-\frac{\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$
Vertical Asymptote	NA	NA	$x = 0, x = \frac{\pi}{2}$
Range	[-3,3]	[-2,2]	$(-\infty, \infty)$

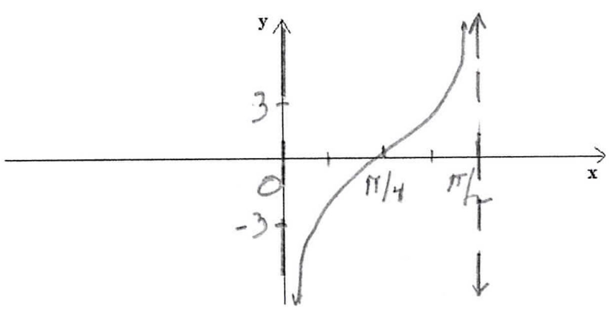
2a.



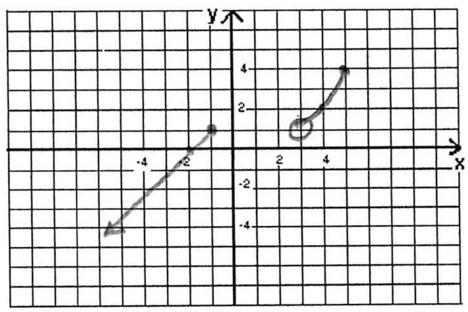
2b.



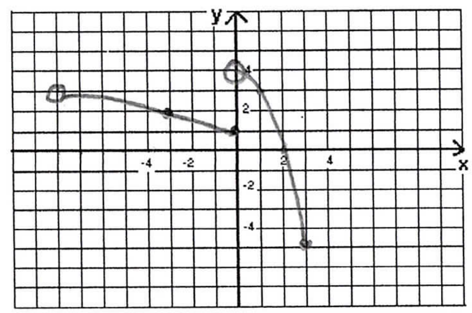
2c.



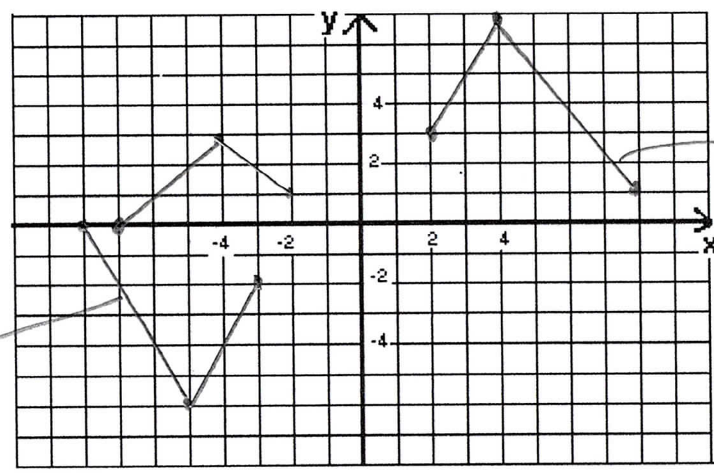
3a.



3b.



4.



4b. "h"

4a. "g"

B. Evaluating Functions, Domains & Inverses

I. 1a. $-\sqrt{3}$ 1b. $-\dfrac{2}{3}$ 1c. -2 1d. Undefined 1e. -3 1f. $-2x$

2a. $(-\infty,-3]\cup[2,\infty)$ 2b. $(-1,\infty)$ 2c. $[2,\infty)$

3. *f(x)* Not 1-1; *g(x)* 1-1 $g^{-1}(x)=3^{-\frac{x}{2}}-1$; h(x) 1-1 $h^{-1}(x)=\{-2,0,3,4\}$

II. $-\dfrac{\pi}{3}$ or -60° **III.** 1. $\dfrac{\sqrt{13}}{7}$ 2. $\dfrac{49}{36}$ 3. $-\dfrac{12\sqrt{13}}{49}$ 4. $\dfrac{-6\sqrt{3}+\sqrt{13}}{14}$

IV. $\dfrac{\sqrt{5}}{3}$ **V.** 1. -4 2. 0.2 3. 3

C. Difference Quotients

1. $-4x-2h-3$ 2. $\dfrac{-3}{(x+h-3)(x-3)}$

D. Solving Equations

1. $-6\ \&\ -4$ 2. $\dfrac{4}{5}$ 3. $\dfrac{3\log 7}{2\log 5-\log 7}$ 4. $\sqrt{13}$ 5. $-5\ \&\ 7$

6. $0,\dfrac{\pi}{6},\pi,\dfrac{5\pi}{6}$

E. Solving Inequalities

I. 1. $(0,2)$ 2. $(-\infty,-3]\cup[1,\infty)$ 3. $(-\infty,-2]\cup[4,\infty)$ 4. $(-\infty,-2)\cup(1,3)$

5. $[-2,1)$

II. $\cos\left(-\dfrac{\pi}{5}\right)$. Angle is in Quadrant IV. In that quadrant, cos positive and sin negative. Therefore cos is larger.

F. Modeling (Applications)

1a. 450 mph 1b. 776 gallons 1c. 900 mph

2. 33-month CD worth $11433; 3-year bond worth $11514; Bond worth more by $81

3a. $A=9LW$ 3b. $A=-2W^2+500W$ 3c. Domain: $(0,250)$ 3d. 31250 sq. ft.

4a. $r=.0676$ 4b. 12.2 grams 4c. Calendar year 2015

5. 1240.1 feet 6. 70 miles 7a. 265 feet 7b. 13 degrees

8. $H(t)=9\sin\left(\dfrac{\pi}{6}t\right)$ 9. Linear: Calendar year 2027; Exponential: 35 fewer years

G. Miscellaneous

1. $(x+1)^2+(y-3)^2=14$ 2. $3+\sqrt{31},\ 3-\sqrt{31}$

3. b+a 4a. 1 4b. $\dfrac{\pi}{4},\dfrac{3\pi}{4},\dfrac{5\pi}{4},\dfrac{7\pi}{4}$ 4c. $\cos^2 x$ 5. QIII; (-0.84,-0.54)