Math 151, Midterm Exam 1 **Review**

Do not assume that the first midterm will be similar to this review sheet. Your first exam may contain questions that do not resemble any of the questions on this review.

Suggested Textbook Problems

Do not assume that merely completing all WebAssign work is sufficient preparation for the midterms. In particular, you will need to show all necessary steps on the exam, not just give an answer. It is strongly recommended that you work out all textbook problems listed on the department website for math 151 (found here).

Other Problems

1. Describe the set $S = \{x \in \mathbb{R} : |2x-4| > 2 \text{ and } |x-3| \le 1\}$ in terms of intervals.

2. Assume that $f(x)$ is a function with domain $\mathbb{R}$, and that $f(x)$ is increasing on $[5,\infty)$. Explain why (a) and (b) must be true:

a) If $f(x)$ is an odd function then $f(x)$ is increasing on $(-\infty,-5]$.

b) If $f(x)$ is an even function then $f(x)$ is decreasing on $(-\infty,-5]$.

3. Complete the square for $2x^2-8x-10$. Use the answer to find the minimum of $2x^2-8x-10$ and to solve $2x^2-8x-10=0$.

4. Find functions $f(x)$ and $g(x)$ with domain $\mathbb{R}$ such that $f\circ g \neq g\circ f$.

5. Find all solutions of $2\sin^2 x = 1+\cos(2x)$ in the interval $[0,2\pi]$.

6. Simplify $\sin^{-1}(\sin(9\pi/4))$, $\sec\left(\sin^{-1} x\right)$, and $\cos(\tan^{-1} x)$.

7. Solve $\ln(x^2+7) = 3\ln 2$.

8. The position of a particle at time t (in seconds) is given by $\dfrac{1}{1+t^2}$ (in feet). Find the average velocity of the particle over the time interval $[1,3]$.

9. Suppose that $f(x)$ is a function such that $f(3+h)-f(3) = e^{2h}-1$ for all $h\neq 0$.

a) Find the slope of the secant line passing through the points $(3,f(3))$ and $(5,f(5))$.

b) Find the slope of the tangent line to the graph of f at $x=3$.

10. One of those scary fish with a light on its head is moving along the x-axis according to the function $s(t) = \dfrac{1}{2}t^2 - 20\sqrt{t}$, where t is measured in minutes, and $s(t)$ is measured in leagues.

a) Compute the *average velocity* of the fish between $t=0$ and $t=4$.

b) Find the *instantaneous velocity* of the fish at time $t=4$. Is the fish moving to the left or to the right at this time?

11. Prove that the equation $x = \cos x$ has a solution.

12. Find the exact values of the following limits. Do not use a calculator. Do not use L'Hôpital's Rule, which appears much later in the textbook.

$$\lim_{x\to 0}\frac{x}{\sin(7x)},\quad \lim_{x\to 0}\frac{\sin(5x)}{\sin(7x)},\quad \lim_{x\to 0}\frac{x}{\tan(x)},\quad \lim_{x\to 0} x\cos\left(x^{-3}\right)$$

$$\lim_{x\to 5^+}\frac{x-5}{|x-5|},\quad \lim_{x\to 5^-}\frac{x-5}{|x-5|},\quad \lim_{x\to 3^+}\frac{x^2-20}{x^2-9},\quad \lim_{x\to 3^-}\frac{x^2-20}{x^2-9}$$

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2+2x-8},\quad \lim_{x\to 2}\frac{x^3-2x^2+x-2}{x^3-x^2-x-2}$$

$$\lim_{x\to 3}\frac{4-\sqrt{5x+1}}{5-\sqrt{8x+1}},\quad \lim_{x\to 3}\frac{4-\sqrt{5x+1}}{6-2x},\quad \lim_{x\to 0}\frac{1-\sec x}{x^2}$$

13. Find constants a, b, and c so that the function $f(x)$ is continuous everywhere.

$$f(x) = \begin{cases} ax^2+b & \text{if } x \le -1 \\ bx+c & \text{if } -1 < x < 1 \\ 2c & \text{if } x = 1 \\ \dfrac{8}{1+x^2} & \text{if } x > 1 \end{cases}.$$

14. Use the ϵ, δ definition of limit to prove $\lim_{x\to 2}(3x+4) = 10$.

15. Find the derivative of each of the following functions using the limit definition of the derivative.

$$f(x) = x^{-2},\qquad g(x) = \sin x,\qquad h(x) = \ln x,\qquad w(x) = \frac{2}{3-x}$$

16. Evaluate

$$\lim_{h\to 0}\frac{e^{7h}-1}{h}$$

by recognizing the limit as the derivative of some function $f(x)$ at a specific point $x = a$.

17. A batter hits a pitched baseball. The height of the baseball is $-16t^2 + 12t + 4$ feet at time t seconds after the bat meets the ball. An outfielder catches the ball when his glove is 6 feet above the ground. At what time did the fielder grab the baseball? What was the maximum height of the ball?

18. A function $f(x)$ is defined by

$$f(x) = \begin{cases} 2x+3 & \text{if } x < 1 \\ 3x+2 & \text{if } x \ge 1 \end{cases}$$

Show that this function is continuous, but not differentiable.

19. Use the limit definition of the derivative to prove that

$$\frac{d}{dx}\cos(ax) = -a\sin(ax)$$

20. Consider the function

$$f(x) = \begin{cases} \sin(x) & \text{if } x < \pi/3 \\ kx & \text{if } x \geq \pi/3 \end{cases}.$$

a) Find a value of k so that f is continuous everywhere.

b) Is there a value of k that makes f *differentiable* at $x = \frac{\pi}{3}$? Justify your conclusion.

21. Find constants a, b such that the function $f(x)$, defined below, is differentiable.

$$f(x) = \begin{cases} 2x + 1 & \text{if } x < 2 \\ ax^2 + b & \text{if } x \geq 2 \end{cases}.$$

22. Find the derivatives of the following functions.

$$\left(x^3 + x\right)^5 (1 + \cos x)^9, \quad \frac{\cot x}{1 + \mathrm{e}^{4x}}, \quad \sin\left(\sqrt{x^4 + x^2 + 3}\right), \quad \csc\left(\mathrm{e}^x + \sqrt{x}\right)$$

23. Find the second derivatives of the following functions.

$$\left(3 + x^{-3}\right)^5, \quad \tan(7x), \quad \frac{1}{\sqrt{\mathrm{e}^x + \cos x}}, \quad \mathrm{e}^{x^2+4x+3}$$

24. Find the first, second, third, and fourth derivatives of $y = \cos(2x)$.

25. Assume $f(x) = \mathrm{e}^{-x^2}$ and $g(x) = \frac{1}{1 + x^2}$. Solve $f''(x) = 0$ and $g''(x) = 0$.

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