

The following is a list of true or false questions from sections 1.1–2.4.

- (1) The empty set can be considered a subspace of a vector space V .
- (2) If S is a subset of a vector space V , then $\text{span}(S)$ equals the intersection of all subspaces of V that contain S .
- (3) If S is a linearly dependent set, and $\mathbf{u}_1, \dots, \mathbf{u}_n$ are vectors in S , then there exist scalars a_1, \dots, a_n , not all zero, such that $a_1\mathbf{u}_1 + \dots + a_n\mathbf{u}_n = \mathbf{0}$.
- (4) If S is a linearly dependent set, then there exist a finite number of vectors $\mathbf{u}_1, \dots, \mathbf{u}_n$ in S , and scalars a_1, \dots, a_n , not all zero, such that $a_1\mathbf{u}_1 + \dots + a_n\mathbf{u}_n = \mathbf{0}$.
- (5) If $S = \{\mathbf{u}_1, \dots, \mathbf{u}_n\}$ is a linearly dependent set of vectors, then there exist scalars a_1, \dots, a_n , not all zero, such that $a_1\mathbf{u}_1 + \dots + a_n\mathbf{u}_n = \mathbf{0}$.
- (6) If S is a linearly independent set, then there exist a finite number of vectors $\mathbf{u}_1, \dots, \mathbf{u}_n$ in S , such that the only scalar a_1, \dots, a_n satisfying $a_1\mathbf{u}_1 + \dots + a_n\mathbf{u}_n = \mathbf{0}$ are $a_1 = \dots = a_n = 0$.
- (7) Subsets of linearly independent sets are linearly independent.
- (8) Subsets of linearly dependent sets are linearly dependent.
- (9) If $\{\mathbf{u}_1, \dots, \mathbf{u}_n\}$ is set of vectors and some vector \mathbf{v} can be expressed as a linear combination of $\mathbf{u}_1, \dots, \mathbf{u}_n$ in more than one way, then $\{\mathbf{u}_1, \dots, \mathbf{u}_n\}$ is linearly dependent.
- (10) A vector space cannot have more than one basis.
- (11) If V is a vector space having dimension n , and if S is a subset of V with n vectors, then S is linearly independent iff S spans V .
- (12) If $T : V \mapsto W$ is a linear transformation, then it carries linearly independent subsets of V into linearly independent subsets of W .
- (13) If $T : V \mapsto W$ is a transformation from V to W , and the only solution to $T(\mathbf{x}) = \mathbf{0}_W$ is $\mathbf{x} = \mathbf{0}_V$, then T is one-to-one.
- (14) If V and W are finite dimensional vector spaces, then $\mathcal{L}(V, W) = \mathcal{L}(W, V)$.
- (15) If V and W are finite dimensional vector spaces, then $\mathcal{L}(V, W)$ and $\mathcal{L}(W, V)$ are isomorphic.
- (16) If V and W are finite dimensional vector spaces, then $\mathcal{L}(V, W)$ and $\mathcal{L}(W, V)$ have the same dimension.
- (17) If a square matrix A satisfies $A^2 = I$, then $A = I$ or $A = -I$.
- (18) If a square matrix A satisfies $A^2 = 0$, where 0 stands for the zero matrix, then $A = 0$.
- (19) If two matrices A and B satisfy $AB = I$, then both A and B are invertible.
- (20) If two matrices A and B satisfy $AB = I$, then $\text{Null}(L_B) = \{\mathbf{0}\}$ and L_A is onto.