Homework 1

January 29, 2009

1.1.6

Show that the midpoint of the line segment joining the points (a, b) and (c, d) is $((a + c)/2, (b + d)/2)$.

The vector pointing from (a, b) to (c, d) is $(c - a, d - b)$, so half of this points from (a, b) to the midpoint of the line segment joining (a, b) and (c, d). Adding this to the first point gives

$$(a, b) + \frac{1}{2}(c - a, d - b) = \left(\frac{a + c}{2}, \frac{b + d}{2}\right).$$

1.2.18

Let $V = \{(a_1, a_2): a_1, a_2 \in \mathbb{R}\}$. For $(a_1, a_2), (b_1, b_2) \in V$ and $c \in \mathbb{R}$, define $(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2)$ and $c(a_1, a_2) = (ca_1, ca_2)$. Is V a vector space over $\mathbb{R}$ with these operations?

(VS 1) does not hold, so V is not a vector space over $\mathbb{R}$ with these operations. For example, $(0, 0) + (1, 1) = (2, 3) \neq (1, 1) = (1, 1) + (0, 0)$.

1.2.22

How many matrices are there in the vector space $M_{m \times n}(\mathbb{Z}_2)$?

These matrices have mn entries, each of which can be one of two values, 0 or 1. Since the locations in the matrix are all distinct, there are $\underbrace{2 \cdot 2 \cdot \cdots \cdot 2}_{mn} = 2^{mn}$ matrices in $M_{m \times n}(\mathbb{Z}_2)$.