Oral Qualifying Exam Syllabus

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1 Committee

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2 Algebraic Topology and Cobordism Theory

2.1 Algebraic Topology

1. The Fundamental Group (the van Kampen theorem, covering spaces)
2. Homology
   (a) Simplicial, singular, and cellular homology
   (b) Simplicial approximation
   (c) Eilenberg-Steenrod homology theories
3. Cohomology
   (a) Simplicial, singular, cellular, de Rham cohomology
   (b) Cohomology with local coefficients, presheaf cohomology
   (c) Cup, cap, and cross products
   (d) Thom isomorphism theorem
   (e) Künneth Formulas
   (f) Poincaré duality, Alexander duality, Lefschetz duality
   (g) Universal Coefficients for Homology
   (h) Eilenberg-Steenrod cohomology theories
4. Homotopy Theory
   (a) Whitehead's theorem
   (b) Cellular approximation theorem, CW approximation theorem
   (c) The Hurewicz theorem
   (d) Fibrations, long exact sequence, Gysin sequence
   (e) Homotopy groups of spheres, stable structure, Freudenthal suspension theorem
   (f) Homotopy construction of cohomology
5. Generalized cohomology and homology theories
   (a) Spectra, Eilenberg-MacLane spectra
   (b) The Brown Representability Theorem
   (c) Steenrod algebra, cohomology operations

2.2 Cobordism Theory

1. h-cobordism theorem
2. (B, f) manifolds and cobordisms
3. Computation of MO_*, MSO_*, MU_*
4. Stiefel-Whitney classes and Pontrjagin classes
5. Oriented cohomology theories

2.3 References

1. Allen Hatcher, *Algebraic Topology.*
2. Raoul Bott; Loring Tu, *Differential Forms in Algebraic Topology.*
3. Davis & Kirk, *Lecture Notes in Algebraic Topology.*
4. Vick, *Homology Theory.*
5. J. P. May, *A Concise Course in Algebraic Topology.*
6. Tom Weston, *An Introduction to Cobordism Theory.*
7. Milnor & Stasheff *Characteristic Classes.*
8. Stong, *Notes on Cobordism Theory.*
9. J. F. Adams, *Stable Homotopy and Generalised Homology*

3 Homological Algebra and Category Theory

3.1 Homological Algebra

1. Chain complexes of modules
2. Derived functors
3. Tor, Ext, and $\lim^1$
4. Universal coefficient theorems
5. Spectral sequences
6. Hyperhomology, Grothendieck spectral sequence, exact couples

3.2 Category Theory

1. Adjoint functors, adjoint functor theorems
2. (Co-)limits
3. Abelian categories
4. Simplicial sets
5. Monoidal categories
6. Model categories

3.3 References

1. Hilton & Stammbach, *A Course in Homological Algebra.*
2. Saunders Mac Lane, *Categories for the Working Mathematician.*
3. Charles Weibel, *An Introduction to Homological Algebra.*
4. J. P. May, *Simplicial Objects in Algebraic Topology.*