

Math 250–Section #4 Quiz #7

Name: _____

1. (6 pts) (a) Find an orthonormal basis for the column space W of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$$

- (b) Use the basis to find the orthogonal projection onto W of the vector

$$v = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}.$$

Answer:

2. (4 pts) (a) If W is a subspace of \mathbb{R}^n , what is the *orthogonal complement* W^\perp of W ? Illustrate with one example.

(b) When $n = 5$ and the dimension of W is 3, explain why the dimension of W^\perp is 2.

Answer:

The routine to obtain a basis that is orthonormal from another basis [Gram–Schmidt process]:

Input basis $S = \{u_1, \dots, u_n\}$

Step 1: Set $v_1 = u_1$

Step 2: Compute v_2, \dots, v_n successively by

$$v_i = u_i - \left(\frac{u_i \cdot v_1}{v_1 \cdot v_1}\right)v_1 - \left(\frac{u_i \cdot v_2}{v_2 \cdot v_2}\right)v_2 - \dots - \left(\frac{u_i \cdot v_{i-1}}{v_{i-1} \cdot v_{i-1}}\right)v_{i-1}$$

Step 3: Set

$$w_i = \frac{v_i}{\|v_i\|}$$

Then $T = \{w_1, \dots, w_n\}$ is an orthonormal basis.