**Math 250–Section #4 Quiz #3**

**Name:** _____

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1. (4 pts) (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}.$$

(b) Use the answer to (a) to solve the system of equations

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 6 \end{bmatrix}.$$

Answer: (a) Using the Gaussian algorithm, we reduce

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 2 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

into

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7/3 & 2/3 & 1 \\ 0 & 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 8/3 & -1/3 & -1 \end{array}\right] = \left[I \mid A^{-1}\right]$$

(b) Multiply both sides of the equation by $A^{-1}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} 9 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -13 \\ -12 \\ 17 \end{bmatrix}.$$

2. (3 pts) [Give explanations using $3 \times 3$ matrices]
(a) What are upper triangular matrices?
(b) If $A$ and $B$ are upper triangular, show that $AB$ is also upper triangular.
(c) If $A$ is upper triangular and invertible, show that $A^{-1}$ is also upper triangular.

Answer: Will discuss in class.

3. (3 pts) Find the $LU$ decomposition of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 4 & -1 & 4 \\ -2 & 1 & 5 \end{bmatrix}$$

Answer:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 4 & -1 & 4 \\ -2 & 1 & 5 \end{bmatrix} \xrightarrow{-2r_1+r_2} \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ -2 & 1 & 5 \end{bmatrix} \xrightarrow{r_1+r_3} \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 6 \end{bmatrix} = U$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-r_1+r_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow{2r_1+r_2} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = L$$

A direct check shows

$$A = LU.$$

Observe the reverse sequence of the related elementary transformations applied to $A$ to get $U$, and applied to $I$ to obtain $L$.

Scratch