Math 250–Section #3   Quiz #1

Name: ____________________

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1. (4 pts) Using the **Gaussian Algorithm**, solve the following system of equations

$$\begin{array}{rcrcrcrcr} x_1 & - & x_2 & + & x_3 & & & = & -4 \\ x_1 & - & x_2 & + & 2x_3 & + & 2x_4 & = & -5 \\ 3x_1 & - & 3x_2 & + & 2x_3 & - & 2x_4 & = & 3 \end{array}$$

Answer the following questions: (a) Is the system consistent? (b) What is the reduced row echelon form of the **augmented** matrix. (c) Which are the basic variables and free variables (if any)? (d) What is the rank and the nullity of the matrix of the **system** [the part involving the variables only]. (e) Describe the solution set in vector form.

Answer: (b) Processing we get

$$\left[\begin{array}{rrrr|r} 1 & -1 & 1 & 0 & -4 \\ 1 & -1 & 2 & 2 & -5 \\ 3 & -3 & 2 & -2 & 3 \end{array}\right] \to \left[\begin{array}{rrrr|r} 1 & -1 & 0 & -2 & -3 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 14 \end{array}\right]$$

(a) The system is NOT consistent
(c) Basic variables: x_1, x_3 [variables in pivot columns]. Free variables: x_2, x_4.
(d) Rank: 2 [number of basic variables]; Nullity: 2 [number of free variables].
(e) N.A.

2. (2 pts) Find the product of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \\ -1 & 0 & 4 \end{bmatrix}$$

by the vector

$$v = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}.$$

What happens if you replace v by $u = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$?

Answer:

$$Av = A \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \\ 13 \end{bmatrix}.$$

The product Au is not possible, the sizes just don't match.

3. (2 pts) (a) What is a **symmetric** matrix? Illustrate with an example.

(b) Argue that the sum $A + B$ of two symmetric matrices is always symmetric.

Answer: (a) A symmetric matrix A is a square matrix equal to its mirror image relative to the diagonal, that is, its entries satisfy $a_{i,j} = a_{j,i}$: Like

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}.$$

We also express this by saying that A is equal to its transpose A^T.

(b) If A and B are symmetric,

$$(A + B)^T = A^T + B^T = A + B,$$

so their sum is also symmetric.

4. (2 pts) Let A be a 3×2 matrix. Consider the system

$$A \cdot \mathbf{x} = \mathbf{0}.$$

Argue that it is always consistent. What are the possible values for the number of free variables? [Explain]

Answer: Such systems, regardless of A, always have the solution

$$\mathbf{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

If we process the system, we may find $0, 1$, or 2 free variables. For example, if A is the null 3×2 matrix, there will be 2 free variables.