

Math 250–Section #4 Hourly #2

Note: Correct answers with justification required!

Name: _____

1. (12 pts) For 2×2 real matrices define and give an example of:
- (a) Invertible
 - (b) Diagonalizable
 - (c) Both symmetric and diagonalizable
 - (d) Non-diagonalizable

Answer:

2. (12 pts) (a) What is the determinant of the matrix

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 7 & 8 \end{bmatrix}$$

(b) If the 5×5 matrix given by its columns $A = [c_1|c_2|c_3|c_4|c_5]$ has determinant 2, find the determinant of the matrix

$$B = [c_2 + c_3|c_3 + c_4|c_4 + c_5|c_5 + c_1|c_1 + c_2].$$

Answer:

3. (15 pts) Given the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix}$$

- (a) Verify that its characteristic polynomial is $-(t - 3)(t - 1)^2$.
- (b) Find bases for each eigenspace.
- (c) Explain how the answer to (b) shows whether A is diagonalizable or not.

Answer:

4. (10 pts) Let A be the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & c \end{bmatrix},$$

where c is some number.

(a) What are the eigenvalues of A ?

(b) If $c \neq 1, 2$, why is A diagonalizable? What happens when $c = 1$ or $c = 2$?

Answer:

5. (14 pts) Find the general solution of the system of differential equations

$$\begin{aligned}y_1' &= -5y_1 + 6y_2 \\y_2' &= -15y_1 + 14y_2\end{aligned}$$

Answer:

6. (15 pts) For the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 & -1 \\ 0 & 0 & 1 & 3 & 2 \\ 2 & 4 & 7 & 0 & 1 \\ 3 & 6 & 11 & 1 & 2 \end{bmatrix}$$

find bases and dimensions for its: (a) column space; (b) row space; (c) null space.

Answer:

7. (12 pts) Let A be a $n \times n$ matrix. Explain the following notions and facts.

- (a) What are eigenvalues and how many eigenvalues can A have?
- (b) If A is invertible, show that no eigenvalue can be 0.
- (c) If 2 is an eigenvalue for A , then 8 is an eigenvalue for the matrix $A^2 + A + 2I$.

Answer:

8. (10 pts) Given the vectors $\mathbf{u} = (-1, 3, 4, 1)$ and $\mathbf{v} = (0, 1, 5, 1)$, find:

(a) $\|\mathbf{u}\|$, $\|\mathbf{v}\|$, $\mathbf{u} \cdot \mathbf{v}$.

(b) Verify that $\mathbf{u} - \mathbf{v} \perp \mathbf{u} + \mathbf{v}$, explain the underlying reason.

Answer:

This page is blank. Use it for scratch.