Math 250–Section #4   Hourly #2

Note: Correct answers with justification required!

Name: ____________________

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1. (12 pts) For 2×2 real matrices define and give an example of:
(a) Invertible
(b) Diagonalizable
(c) Both symmetric and diagonalizable
(d) Non-diagonalizable

Answer:

2. (12 pts) (a) What is the determinant of the matrix

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 7 & 8 \end{bmatrix}$$

(b) If the 5×5 matrix given by its columns $A = [c_1|c_2|c_3|c_4|c_5]$ has determinant 2, find the determinant of the matrix

$$B = [c_2 + c_3|c_3 + c_4|c_4 + c_5|c_5 + c_1|c_1 + c_2].$$

Answer:

3. (15 pts) Given the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix}$$

(a) Verify that its characteristic polynomial is $-(t-3)(t-1)^2$.
(b) Find bases for each eigenspace.
(c) Explain how the answer to (b) shows whether A is diagonalizable or not.

Answer:

4. (10 pts) Let A be the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & c \end{bmatrix},$$

where c is some number.
(a) What are the eigenvalues of A?
(b) If $c \neq 1, 2$, why is A diagonalizable? What happens when $c = 1$ or $c = 2$?

Answer:

5. (14 pts) Find the general solution of the system of differential equations

$$\begin{aligned} y_1' &= -5y_1 + 6y_2 \\ y_2' &= -15y_1 + 14y_2 \end{aligned}$$

Answer:

6. (15 pts) For the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 & -1 \\ 0 & 0 & 1 & 3 & 2 \\ 2 & 4 & 7 & 0 & 1 \\ 3 & 6 & 11 & 1 & 2 \end{bmatrix}$$

find bases and dimensions for its: (a) column space; (b) row space; (c) null space.

Answer:

7. (12 pts) Let A be a $n \times n$ matrix. Explain the following notions and facts.
(a) What are eigenvalues and how many eigenvalues can A have?
(b) If A is invertible, show that no eigenvalue can be 0.
(c) If 2 is an eigenvalue for A, then 8 is an eigenvalue for the matrix $A^2 + A + 2I$.

Answer:

8. (10 pts) Given the vectors $\mathbf{u} = (-1, 3, 4, 1)$ and $\mathbf{v} = (0, 1, 5, 1)$, find:
(a) $||\mathbf{u}||$, $||\mathbf{v}||$, $\mathbf{u} \cdot \mathbf{v}$.
(b) Verify that $\mathbf{u} - \mathbf{v} \perp \mathbf{u} + \mathbf{v}$, explain the underlying reason.

Answer:

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