

## Comments

$$
\begin{array}{lll}
\text { Restrictions : } & (\exists x \in A) P(x) \text { means: } & (\exists x)[(x \in A) \wedge P(x)] \\
& \text { while }(\forall x \in A) P(x) \text { means: } & (\forall x)[(x \in A) \rightarrow P(x)]
\end{array}
$$

$$
\begin{aligned}
\text { Hence } & \sim(\forall x \in A) P(x) \equiv(\exists x \in A) \sim P(x) \\
\text { while } & \sim(\exists x \in A) P(x) \equiv(\forall x \in A) \sim P(x)
\end{aligned}
$$

Remark. The predicate $(\forall x)[Q(x) \rightarrow R(x)]$ is usually (and somewhat sloppily?) written as "if $Q(x)$ then $R(x)$ ". When we use the words "if $\ldots$, then ..." (rather than the symbol $\ldots \rightarrow \ldots$ ), the universal quantifier is tacitly understood.
Thus, $(\forall x \in A) P(x)$ means: if $x \in A$ then $P(x)$.
set union: $\quad(\forall x)[(x \in A \cup B) \leftrightarrow(x \in A$ or $x \in B)]$
set intersection: $\quad(\forall x)[(x \in A \cap B) \leftrightarrow(x \in A$ and $x \in B)]$
set difference: $\quad(\forall x)[(x \in A \backslash B) \leftrightarrow(x \in A$ and $x \notin B)]$
symmetric difference: $\quad(\forall x)[(x \in A \circ B) \leftrightarrow(\{x \in A$ and $x \notin B\}$ or $\{x \in B$ and $x \notin A\})]$

Containment:
subset $\quad A \subset B \quad[$ or $A \subseteq B]: \quad(\forall x \in A) x \in B$, that is, if $x \in A$ then $x \in B$ - read: " $A$ is a subset of $B$ ".
superset $A \supset B[$ or $A \supseteq B]-$ " $A$ is a superset of $B$ ", or " $A$ contains $B$ "
Note that $A \cup B$ is a set, while $A \subset B$ is a statement.

