A completely inefficient primality test (Wilson's theorem)

Theorem. *Let $p \in \mathbb{N}$, $p \geq 2$.*

(*) If p is prime, then $(p-1)! \equiv -1 \pmod{p}$.

(**) If p is composite, then $(p-1)! \equiv 0 \pmod{p}$.

Proof.

(**) is trivial.

For (*), prove and use the following simple lemma.

Lemma. *Let G be a finite group with identity e. Then,*

$$\prod_{g \in G} = \prod_{\substack{g \in G \\ g^2 = e}}$$

(Find both places in the proof where the fact that p is prime is used.)