$O(n)$ denotes the orthogonal group over $\mathbb{R}^n$ (the multiplicative group of real orthogonal $n \times n$ matrices), and $SO(n)$ (special orthogonal group) the subgroup of $O(n)$ of matrices with determinant 1. An isometry of $\mathbb{R}^n$ is a distance preserving map from $\mathbb{R}^n$ to $\mathbb{R}^n$.

Theorem. *The orthogonal group $O(n)$ is isomorphic to the group of all isometries of $\mathbb{R}^n$ which leave the origin fixed. The special orthogonal group $SO(n)$ is isomorphic to the group of all rotations of $\mathbb{R}^n$.*

The first part of the claim follows from the following step-by-step analysis.

Lemma. *Let T be an isometry of $\mathbb{R}^n$ which leaves the origin fixed ($T(\mathbf{0}) = \mathbf{0}$). Then,*

1. T is a linear map: $(\forall \mathbf{x}, \mathbf{y} \in \mathbb{R}^n)(\forall c, d \in \mathbb{R}) T(c\mathbf{x} + d\mathbf{y}) = cT(\mathbf{x}) + dT(\mathbf{y})$.

2. T preserves angles.

3. T preserves dot products.

4. For the matrix A of T in the standard basis we have $A^*A = AA^* = I$ (orthogonal matrix).

5. $\det(A)$ is either +1 or -1.

For the second part of the theorem: Let $A \in O(n)$. Then,

$$\det(A - I)\det(A^*) = \det(I - A^*) = \det(I - A) = (-1)^n \det(A - I)$$

that is, either $\det(A) = (-1)^n$, or 1 is an eigenvalue of A (A fixes a line).

Similarly,

$$\det(A + I)\det(A^*) = \det(I + A^*) = \det(I + A)$$

that is, either $\det(A) = 1$, or -1 is an eigenvalue of A (A reverses a line).

In summary:

when n is odd:
if $A \in SO(n)$, then 1 is an eigenvalue of A.
if $A \notin SO(n)$, then -1 is an eigenvalue of A.

when n is even:
if $A \notin SO(n)$, then both 1 and -1 are eigenvalues of A.

Remark: Since an orthogonal matrix preserves length, it can only have 1 and -1 as its (perhaps multiple) real eigenvalues, but it need not have a complete set of eigenvectors (e.g., a rotation by 20 degrees in $\mathbb{R}^2$ has no real eigenvalues at all).

For more information see

http://www-history.mcs.st-and.ac.uk/~john/geometry/Lectures/L4.html