Homework 6
Math 485
November 6, 2015

In all of the following questions, suppose $S_t$ follow the Black-Scholes model under a risk neutral measure $Q$:

$$dS_t = rS_t dt + \sigma S_t dW_t.$$

1. Derive the Black-Scholes formula. Try to go as far as you can before consulting the notes or textbook.

2. Compute $V_0$ for the following Euro-style derivatives:
   a. $V_T = (S_T^\beta - K)^+, \beta$ a constant.
   b. $V_T = 1$ if $S_T < K$, $V_T = 0$ otherwise.
   c. $V_T = S_T^\beta$, $\beta$ a constant.
   d. $V_T = \log(S_T)$.
   e. $V_T = 1$ if $K_1 < S_T < K_2$, $V_T = 0$ otherwise.
   f. $V_T = (K - S_T^\beta)^+, \beta$ a constant.

3. Let $\rho = \frac{\partial V_0}{\partial r}$ where $V_T = (S_T - K)^+$. Prove that

$$\rho = KTe^{-rT}N(d_2),$$

where

$$d_2 = \frac{(r - \frac{1}{2}\sigma^2)T - \log(K/S_0)}{\sigma \sqrt{T}}$$