1. John, your friend, tells you that he has two daughters. You also know that he has three children in total. What, then, is the probability that his youngest child is a girl? (Assuming that a boy and a girl are equally likely).

**Ans:** \( P(\text{youngest G}|2\text{G}) = \frac{2}{3} \), since out of the possible outcomes

\[ GGB, GBG, BGG, \]

2 of them give youngest \( G \), and they are all equally likely.

2. The arrival time of Rutgers shuttles to the Hill Center's bus stop from 5:00pm to 5:30pm on a weekday is uniformly distributed. In other words, let \( X \) be the waiting time (in minutes) after 5:00 pm until the arrival of a particular shuttle, then \( X \) has the p.d.f

\[
f_X(x) = \frac{1}{30}, 0 \leq x \leq 30. \tag{1}
\]

Suppose there are 30 shuttles arriving at the Hill Center from 5:00 pm to 5:30 pm, and their distribution are i.i.d. What, then, is the approximate probability that their average arrival time on a particular weekday is before 5:12 pm?

**Ans:** Let \( X_i, 1 \leq i \leq 30 \) be i.i.d. with uniform \([0,30]\) distribution. Then \( \mu := E(X_i) = 15 \) and \( \sigma^2 := Var(X_i) = 300 - 15^2 = 75 \). By the CLT,

\[
\frac{\sqrt{n}(\overline{X} - \mu)}{\sigma} \approx N(0,1),
\]

or

\[
\overline{X} \approx Z \frac{\sigma}{\sqrt{n}} + \mu,
\]
where \( Z \) has \( N(0,1) \) distribution. So
\[
P(\bar{X} \leq 12) \approx P(Z \leq \frac{(12 - \mu)}{\sigma} \sqrt{n}) \\
\approx P(Z \leq \frac{-3\sqrt{30}}{\sqrt{75}}) = P(Z \leq -3\frac{\sqrt{2}}{5})
\]

3. Completing the Financial Math program at Rutgers has a positive effect on students’ placement. A student who has succesfully completed the program has a 70% probability of landing a job with Goldman Sachs. A student who did not succesfully complete the program, however, only has a 40% probability of landing such a job. Your friend, Tom, just got a position with Goldman Sach. Suppose the probability of a student succesfully completing the Financial Math program at Rutgers is 80%. What is the probability that Tom succesfully finished his Financial Math program at Rutgers?

Ans:
(Draw a probability tree to illustrate this situation). The answer is
\[
\frac{(0.8)(0.7)}{(0.8)(0.7) + (0.2)(0.4)} = 0.875
\]

Approach using Bayes’ formula: Let \( A \) be the event that a person successfully completed the Financial Math program. Then \( P(A) = 0.8 \). \( B \) be the event that a person lands a job with Goldman. Then \( P(B|A) = 0.7 \) and \( P(B|A^C) = 0.4 \). We are asked to find \( P(A|B) \). By Baye’s formula:
\[
P(A|B) = \frac{P(B|A)P(A)}{P(B)}.
\]
Now \( P(B) = P(B|A)P(A) + P(B|A^C)P(A^C) = (0.7)(0.8) + (0.4)(0.2) \). And so the answer is \( \frac{(0.8)(0.7)}{(0.8)(0.7) + (0.2)(0.4)} = 0.875 \).

4. The breakdown time of the Apple Ipad is an exponential(5) random variable. In other words, let \( X \) be the time until break down (in years) of a particular Ipad, then \( X \) has the p.d.f
\[
f_X(x) = \frac{1}{5}e^{-\frac{x}{5}}, 0 \leq x < \infty.
\]
Suppose an Apple store has 30 Ipads, and the distribution of their break down times are i.i.d. What, then, is the approximate probability that the average break down time of these Ipads is before 2 years?

**Ans:** One can check that $E(X) = 5$ and $Var(X) = 25$. Therefore $\bar{X}$ has distribution $N(5, \frac{25}{30})$. So

$$P(\bar{X} \leq 2) = P(Z \leq \frac{2 - 5}{\sqrt{\frac{25}{30}}}) \approx P(Z \leq -3.28) \approx 0.$$ 

5. There are 120 students in a Calculus 1 class at Rutgers. Suppose that each student has a probability of .3 of getting an A in this class, and the students’ performance is independent of one another. What, then, is the approximate probability that the class will have at least 20 students getting an A?

**Ans:** Let $X = Bin(.3, 120)$. Then $E(X) = 36$ and $Var(X) = 25.2$. by the normal approximation to the Binomial we have

$$P(X \geq 20) \approx P(Z \geq \frac{20 - 36}{\sqrt{25.2}}) \approx P(Z \geq -3.18) \approx 1.$$ 

$\blacksquare$