1. (10 pts) Determine the value(s) of \( c \) for which the given matrix is not invertible.

\[
\begin{bmatrix}
1 & 2 & -1 \\
2 & 3 & c \\
0 & c & -15
\end{bmatrix}
\]

Ans: (Expanding along column 1) The determinant is:

\[
1(3(-15) - c^2) - 2(2(-15) + c) = -c^2 - 2c + 15.
\]

We require the determinant equals to 0, i.e. \(-c^2 - 2c + 15 = 0\), and by factoring

\[(c + 5)(3 - c) = 0.\]

So \(c = -5, 3\).

2. (10 pts) Find the standard matrix associated with the linear transformation \( T \) and use it to determine whether \( T \) is onto.

\( T : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) defined by

\[
T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} 3x_2 \\ 2x_1 - x_2 \\ x_1 + x_2 \end{bmatrix}
\]

Ans: The standard matrix is

\[
A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \\ 1 & 1 \end{bmatrix}.
\]

\( T \) is onto iff \( A \) has rank 3. However, it is clear that \( rank(A) \leq 2 \). Therefore, \( T \) is not onto.