MATHEMATICS 503 FALL 2014

Instructor: H. J. Sussmann

Exercises for the sixth week of classes, i.e., for the lectures of October 7 and 9.

You should practice by trying to do as many problems as you can from Chapter III of the book.

PROBLEM 1. Book, Page 93, Problem 2. Additional question: What happens if U is unbounded? (NOTE: The *boundary* of U is the set $\mathrm{Clos}\, U \backslash U$.)

PROBLEM 2. Book, Page 93, Problem 3.

PROBLEM 3. Book, Page 102, Problem 1.

PROBLEM 4. Book, Page 102, Problems 3 and 4.

PROBLEM 5. Book, Page 103, Problems 7 and 8.

PROBLEM 6. Book, Page 103, Problem 11.

PROBLEM 7. An open subset U of $\mathbb{C}$ is *holomorphically simply connected* if every holomorphic function $f : U \mapsto \mathbb{C}$ has a holomorphic primitive on U. (We will prove later that U is holomorphically simply connected if and only if it is simply connected.)

I. Prove that if U, V are holomorphically simply connected open subsets of $\mathbb{C}$ and $U \cap V$ is connected, then $U \cup V$ is holomorphically simply connected.

II. Give an example of two connected holomorphically simply connected open subsets U, V such that $U \cap V$ is not connected and $U \cup V$ is not holomorphically simply connected.

PROBLEM 8. Book, Page 118, Problem 1.