MATHEMATICS 503 FALL 2014

Instructor: H. J. Sussmann

Exercises for the third week of classes, i.e., for the lectures of September 16 and 18.

You should practice by trying to do all the problems in Chapter 2 of the book.

PROBLEM 1. Book, Page 58, Problem 4.

PROBLEM 2. Book, Page 59, Problem 5.

PROBLEM 3. Book, page 60, Problem 12.

PROBLEM 4. Book, Page 68, Problem 2. (One of the two identities of Part (a) was proved in class. You should do the other one.)

PROBLEM 5. Book, Page 68, Problem 5.

PROBLEM 6. Book, Page 75, Problems 4 and 5.

PROBLEM 7. Prove that if f is an analytic function on an open subset U of $\mathbb{C}$, $z_0 \in U$, and $f(z_0) \neq 0$, then there exists an analytic function $g : V \mapsto \mathbb{C}$, defined on some open set V such that $z_0 \in V \subseteq U$, such that

$$e^{g(z)} = f(z) \qquad \text{for all} \quad z \in V\,.$$

(Note: Such a function is called a "branch of $\ln f(z)$".)

PROBLEM 8. Let U be the open set $\{z \in \mathbb{C} : 1 < |z| < 2\}$. Conclude from the result of Problem 8 that for every $z_0 \in U$ there exists an analytic function $g : V \mapsto \mathbb{C}$, defined on an open neighbborhood V of z_0 such that $V \subseteq U$, such that

$$e^{g(z)} = z \qquad \text{for all} \quad z \in V\,.$$

And then prove that there does not exist an analytic function $g : U \mapsto \mathbb{C}$ such that

$$e^{g(z)} = z \qquad \text{for all} \quad z \in U\,.$$

(In other words: a branch on $\ln z$ exists locally, near every point of U, but there does not exist a global branch, defined on all of U.)