MATHEMATICS 503 FALL 2014

Instructor: H. J. Sussmann

Exercises for the Thanksgiving week.

QUESTION 1. Prove that if U is an open subset of $\mathbb{C}$, a, b are real numbers such that $a < b$, f_t is a holomorphic function on U for each $t \in [a, b]$, and the function $f : U \times [a, b] \mapsto \mathbb{C}$ given by $f(z, t) = f_t(z)$ is continuous, then

(1) the function $F : U \mapsto \mathbb{C}$ given by
$$F(z) = \int_a^b f(z,t)dt$$
is holomorphic.

(2) The function $g : U \times [a, b] \mapsto \mathbb{C}$ given by $g(z, t) = f_t'(z)$ is continuous.

(3) The derivative F' of F is given by
$$F'(z) = \int_a^b g(z,t)dt\,.$$

(NOTE: You don't necessarily have to prove (1) first, then (2), then (3). It could be that it is more convenient to prove them in a different order. For example, you may[1] choose to prove (2) first.)

QUESTION 2. Book, Page 83, Problems 1, 2, 3, 4, 5. (NOTE: "Analytic isomorphism" means the same as "biholomorphic function.")

QUESTION 3. 172, Problems 11, 12. (The *order* of a point z for a meromorphic fiunction f is n if z is a zero of f of order n, $-n$ if z is a pole of f of order n, and 0 if z is neither a zero nor a pole of f.)

QUESTION 4. Book, Page 213, Problem 1.

QUESTION 5. Book 159, 7, 8, 9.

[1]But you don't *have* to.