

Answers to homework problem “che1”

We are given this chemotaxis equation (one space dimension) for the concentration of a microorganism (assuming no additional reactions, transport, etc):

$$\frac{\partial c}{\partial t} = \frac{\partial c}{\partial x} \frac{(2x-6)}{(2+(x-3)^2)^2} - 2c \left(\frac{(2x-6)^2}{(2+(x-3)^2)^3} - \frac{1}{(2+(x-3)^2)^2} \right).$$

- (1) What is the potential function? (Give a formula for it.)
- (2) Where (at $x = ?$) is the largest amount of food?

Answer:

- (1) The chemotaxis equation in one dimension (assuming no other effects) is:

$$\frac{\partial c}{\partial t} = -\alpha \frac{\partial c}{\partial x} V' - \alpha c V''$$

so we match the above with this:

$$V_x = -\frac{2(x-3)}{(2+(x-3)^2)^2}$$

So you only need to integrate to find V . I would think that it is clear just looking at it that we can pick

$$V(x) = (2+(x-3)^2)^{-1}$$

but if it is not clear, then you can substitute $u = 2+(x-3)^2$ and find $-\int du/u^2 = 1/u$.

- (2) The maximum of V is obtained when the denominator is minimized, that is to say at $x = 3$, so this is the location of the food source. (If you did not integrate, you can also notice that $V_x = 0$ means that $x = 3$.)