In all problems, $\lambda = \ln 2/5$ (because $e^{5\lambda} = 2$), so the $e^{\lambda t}$ term will be $e^{(\ln 2/5)t} = 2^{t/5}$. The general solution is $2^{t/5} f(x-4t)$.

1. $2^{0/t} f(x-0) = 1$ means $f(u) = 1$ so

$$c(x,t) = 2^{t/5}.1 = 2^{t/5}.$$

2. $2^{0/t} f(x-0) = 2 + \cos x$ means $f(u) = 2 + \cos u$ so

$$c(x,t) = 2^{t/5}(2 + \cos(x-4t)).$$

3. $2^{0/t} f(x-0) = \frac{1}{1+x^2}$ means $f(u) = \frac{1}{1+u^2}$ so

$$c(x,t) = 2^{t/5}\frac{1}{1+(x-4t)^2}.$$

4. $2^{1/5} f(x-4) = 2 + \cos x$ means (substitute $u = x-4$ so $x = u+4$) that $f(u) = 2^{-1/5}(2 + \cos(u+4))$ so

$$c(x,t) = 2^{(t-1)/5}(2 + \cos(x+4-4t)).$$

5. $2^{t/5} f(-4t) = 1$ means (substitute $u = -4t$ so $t = -u/4$) that $f(u) = 2^{u/20}$ so

$$c(x,t) = 2^{t/5}(2^{(x-4t)/20}) = 2^{x/20}.$$

6. $2^{t/5} f(-4t) = \sin t$, same substitution, gives

$$c(x,t) = -2^{x/20} \sin\left(\frac{x-4t}{4}\right).$$

7. $2^{t/5} f(1-4t) = \frac{1}{1+e^t}$ and substitution $u = 1-4t$, so $t = (1-u)/4$, gives

$$c(x,t) = -2^{(x-1)/20}\frac{1}{1+e^{\frac{1-x+4t}{4}}}.$$