

## TRIG IDENTITIES

Periodicity: if  $f$  is any of the six trig functions, then for all  $x \in \mathbb{R}$ ,  $f(x + 2\pi) = f(x)$ .

Definitions in terms of sine and cosine:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}, \quad \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

Reflection identities:

$$\sin(-\theta) = -\sin \theta, \quad \cos(-\theta) = \cos \theta, \quad \tan(-\theta) = -\tan \theta$$

Rotation identities:

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos \theta, \quad \cos\left(\theta + \frac{\pi}{2}\right) = -\sin \theta, \quad \tan\left(\theta + \frac{\pi}{2}\right) = -\cot \theta$$

$$\sin(\theta + \pi) = -\sin \theta, \quad \cos(\theta + \pi) = -\cos \theta, \quad \tan(\theta + \pi) = \tan \theta$$

Pythagorean identities:

$$\sin^2 \theta + \cos^2 \theta = 1, \quad \tan^2 \theta + 1 = \sec^2 \theta, \quad 1 + \cot^2 \theta = \csc^2 \theta$$

Addition formulas:

$$\sin(x + y) = \sin x \cos y + \sin y \cos x, \quad \sin(x - y) = \sin x \cos y - \sin y \cos x$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y, \quad \cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}, \quad \tan(x - y) = \frac{\tan x - \tan y}{1 - \tan x \tan y}$$

Double-angle formulas (follow from addition formulas):

$$\sin 2\theta = 2 \sin \theta \cos \theta \quad \cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta, \quad \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Half-angle formulas (follow from double-angle formulas):

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}, \quad \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$