

## STUDY GUIDE FOR FIRST MIDTERM

The first midterm will be held in class on Monday, October 19th. It will cover Chapters 1 and 3. (Notice that the section on determinants will not be covered on this exam). Anything discussed in lecture or in the text is fair game for the exam. The questions will more or less resemble the homework questions. Be advised that you will not be expected to reproduce any “proofs,” although there will be several questions of a rather theoretical nature. Here is a not-necessarily-comprehensive list of topics to study while preparing for the exam:

- (1) Basic facts and terminology concerning systems of linear equations
  - Know how to tell whether or not a linear system is consistent.
  - Be able to solve a consistent linear system and express the solution set parametrically.
  - Know the three elementary row operations.
  - Understand the connection between linear systems and the matrix-vector equations  $A\vec{x} = \vec{b}$ .
  - Know the basic facts about homogeneous linear systems, including the relationship between  $A\vec{x} = \vec{b}$  and  $A\vec{x} = \vec{0}$ .
- (2) Gaussian elimination
  - Be able to do it!
  - Know the definitions of REF and RREF, as well as basic facts about the existence and uniqueness of these special matrix forms.
  - Be familiar with elementary matrices, and their connection with elementary row operations.
  - The equation  $PA = R$ . Know how to find  $P$ , and know the theorem that guarantees  $P$  exists.
- (3) Basic matrix algebra
  - Know all the basic facts about how to scale, add, and multiply matrices, take powers, etc.
  - Be familiar with the various special kinds of matrices — eg, square, diagonal, triangular, scalar, symmetric, row, column, identity.
  - Be familiar with special matrix operations such as trace and transpose.
- (4) Matrix invertibility
  - Be able to tell whether or not a matrix is invertible, and if it is, know how to find the inverse.
  - Know the matrix invertibility theorem (1.6.4 in the text).
  - Know that certain kinds of matrices are invertible — eg, elementary matrices, products of invertible matrices, the transpose of an invertible matrix.
  - Know the formula for inverting a  $2 \times 2$  matrix.
- (5) Vectors in  $\mathbb{R}^3$ 
  - Be familiar with their basic algebraic properties.
  - Magnitude of a vector, distance between two points, triangle inequality.
  - The dot product, and how it relates to lengths of vectors and angles between vectors.
  - Be able to compute vector projections.
  - Be able to compute cross products, and know the basic algebraic properties of cross products.
  - Be familiar with the standard unit vectors  $\vec{e}_1, \dots, \vec{e}_n$  in  $\mathbb{R}^n$ . Remember that for any matrix  $A$ ,  $A\vec{e}_j$  is the  $j$ th column of  $A$  and  $\vec{e}_i^T A$  is the  $i$ th row of  $A$ .