

## HAPPY BIRTHDAY PROBLEMS

- (1) The trace of any skew-symmetric matrix.
- (2) The number of possible values for the rank of the matrix  $\vec{x}\vec{y}^T$ , given only that  $\vec{x}$  and  $\vec{y}$  are column vectors in  $\mathbb{R}^8$ .
- (3) The distance between the parallel planes  $3x - 4y + 12z = 21$  and  $3x - 4y + 12z = -5$ .
- (4) The dimension of  $\ker T$ , where  $T$  is the differentiation operator on the vector space  $P_\infty$  of all polynomials.

- (5) The dimension of the eigenspace of  $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$  corresponding to the eigenvalue  $\lambda = 2$ .

- (6) The least possible nullity of  $A^T$  given that  $A$  is a  $43 \times 34$  matrix.

- (7) The largest possible rank of a linear transformation  $T: \mathbb{R}^{10} \rightarrow \mathbb{R}^9$  such that  $T(\vec{x}) = \vec{0}$  for all  $\vec{x} \in V$ , where

$$V = \{\langle v_1, \dots, v_{10} \rangle \in \mathbb{R}^{10} \mid v_1 + 3v_4 = 0 \text{ and } 2v_4 - v_6 + v_7 = 0\} \leq \mathbb{R}^{10}.$$

- (8) The number of distinct (real) eigenvalues of the matrix  $A = \begin{bmatrix} -2 & 1 & 0 & 0 \\ -5 & 2 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ .