HANDOUT: GAUSSIAN ELIMINATION

Recipe for solving a system of linear equations: first find the augmented matrix of the system, and then perform a sequence of computations on this matrix, called *elementary row operations*, to change the matrix into certain special forms that make it easy to solve the system. The algorithm for transforming the matrix into these special forms is called *Gaussian elimination.* The special forms we transform the matrix into are called *row echelon form (REF)* and *reduced row echelon form (RREF)*. There are three elementary row operations:

- Interchange any two rows of the matrix. *(row interchange)*
- Multiply every entry of some row of the matrix by the same nonzero constant. *(scaling)*
- Add a multiple of one row of the matrix to another row. *(row addition)*

Basic fact about elementary row operations: performing an elementary row operation on the augmented matrix of a system of linear equations produces the augmented matrix of an equivalent system of linear equations, ie, a system with the same solution set.

Key theorem about RREF: Every matrix can be transformed into one and only one matrix in RREF by means of a sequence of elementary row operations.

Once the augmented matrix has been put into RREF, check to see if the system is consistent, and, if so, solve for the leading variables in terms of the free variables to obtain a parametrically defined *general solution* to the system.

GAUSSIAN ELIMINATION: method for transforming a matrix into its RREF form.[1]

Step 1 Locate the leftmost nonzero column. This is a pivot column. The pivot position is at the top of this column.

Step 2 Select a nonzero entry in the pivot column as a pivot. If necessary, interchange rows to move this entry into the pivot position.

Step 3 Scale the top row to make the pivot a 1.

Step 4 Repeatedly use the row addition operation to create zeros in all positions below the pivot.

Step 5 Now ignore the row containing the pivot position and all rows, if any, above it. Apply steps 1-4 to the sub-matrix that remains. Repeat the process until there are no more nonzero rows to modify.

These five steps have transformed the matrix into REF. Use step 6 to transform it into RREF.

Step 6 Beginning with the rightmost pivot, and working upward and to the left, create zeros above each pivot.

[1] If we use only the first five steps, this procedure produces a matrix in REF and is called *Gaussian elimination*; carrying the procedure through to the sixth step and producing a matrix in RREF is sometimes called *Gauss-Jordan elimination.* For large linear systems, using Gaussian elimination together with "back-substitution" in order to solve the system is about 50% more efficient that using Gauss-Jordan elimination.