

OPTIMIZATION AND RELATED RATES

OPTIMIZATION PROBLEMS

- (1) A farmer has 2400 feet of fencing and wants to fence off a rectangular field along a straight river so as to make two rectangular enclosures of equal size, each bordering the river. He needs no fence along the river. What are the dimensions of the largest possible field?
- (2) Find a positive number such that the sum of the number and its reciprocal is as small as possible.
- (3) A box with an open top is to be constructed from a square piece of cardboard 3 feet wide by cutting out a square from each corner and bending up the sides. Find the largest volume such a box can have.
- (4) A right circular cylinder is inscribed in a cone with height h and base of radius r . Find the largest possible volume of such a cylinder.
- (5) A right circular cylinder is inscribed in a sphere of radius r . Find the largest possible surface area of such a cylinder.

RELATED RATES PROBLEMS

- (1) Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s . How fast is the radius of the balloon increasing when the diameter is 50 cm?
- (2) A plane flying horizontally one mile up at 500 mph passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station.
- (3) A street light is mounted atop a pole that is 15 feet high. A 6 foot tall man walks away from the pole at 5 ft/s. How fast is the tip of his shadow moving when he is 40 feet from the pole?
- (4) Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min to form a cone-shaped pile whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 feet high?
- (5) The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min . At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 10 cm^2 ?