STUDY GUIDE FOR FINAL EXAM

The final exam will be cumulative, covering everything that was presented in lecture together with Chapters 1 through 5 of the textbook *excluding* sections 1.8, 4.9, 4.11, 4.12, and 5.9. Aside from the preliminary topics of Chapter 1, which will not be emphasized directly, this material will appear on the exam in relatively equal proportions. The problems will be similar to those on the three midterms and from the homework; there will as usual be a blend of theory and computation. In particular, you *will be responsible* for knowing the theory of limits as it was presented in Chapter 2.

I will post a list of practice problems for the final exam on the course website over the weekend, and hand out a hard copy in class on Monday. In the meantime, this review sheet will serve as a list of topics to study. I will not list topics from Chapter 1; but as you have no doubt already realized, the single most important skill you can acquire for understanding calculus and doing well in this class (and on the final) is *graphing*: in particular the ability to visualize and interpret algebraic information from the graphs of a wide range of commonly occurring functions.

Chapter 2

- The precise definitions of (the many various kinds of) limits
  - limits from the left and right
  - limits at $\pm\infty$
  - limits equal to $\pm\infty$
  - vertical and horizontal asymptotes
- How to prove that a limit exists, or is equal to some value L
- How to prove that (or recognize when) a limit does not exist
- The limit laws (eg, "the limit of the sums is the sum of the limits," etc)
- Evaluating limits (eg, the exercises of Section 2.3)
- The definition of the continuity of a function at a point, and on an interval
- Continuity from the left / right
- Examples of continuous / non-continuous functions
  - polynomials are everywhere continuous
  - rational functions are continuous on their domains
  - the trig functions are continuous on their domains
  - $y = \frac{|x|}{x}$ and $y = \sin \frac{1}{x}$ are not continuous at $x = 0$
- The various kinds of discontinuity (removable, essential, jump, infinite)
- Theorems involving continuity (eg, Theorems 2.4.2, 2.4.4 in the textbook)
- The Squeeze Theorem
- The Intermediate Value Theorem (EVT) and the Extreme Value Theorem (EVT)
- The limits $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$ and $\lim_{x\to 0} \dfrac{\cos x - 1}{x} = 0$
- Evaluating trig limits (eg, problems 1 – 30 in Section 2.5)

Chapter 3

- Secant lines and difference quotients
- The algebraic definition of the derivative: $f'(a) = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$
  - Computing a derivative directly from the definition
- The geometric interpretation of the derivative: $f'(a)$ is the slope of the line tangent to the curve $y = f(x)$ at the point $(a, f(a))$
  - An equation of the line tangent to the curve $y = f(x)$ at the point $(a, f(a))$
- Interpreting the derivative as a rate of change (eg, as a velocity)
- Leibniz notation for derivatives

- Differentiability implies continuity
- The differentiation rules (eg, product rule, quotient rule, chain rule)
- Differentiation of:
  - polynomials
  - rational power functions (eg, $y = x^{1/3}$)
  - trig functions
  - rational functions
- Higher-order derivatives (including patterns in the derivatives of sine and cosine)
- Implicit differentiation

Chapter 4

- Absolute (ie, global) and relative (ie, local) extrema
- Fermat's Theorem
- Critical points
- Procedure for finding the absolute extrema of a continuous function on a closed interval
- Rolle's Theorem and the Mean Value Theorem (MVT)
- Applications of the Mean Value Theorem (eg, Theorem 4.2.4 in the text)
- Increasing / decreasing functions and Theorem 4.2.3 in the text
- The First and Second Derivative Tests
- Concavity and inflection points
- Asymptotes, vertical tangents, and cusps
- Graphing! (Section 4.8)
- Optimization (Section 4.5)
- Related rates (Section 4.10)

Chapter 5

- Summation notation
- The Antidifferentiation Problem and its solution via *area*
  - the "area-so-far" function
- Riemann sums (definitions and examples)
- The definite integral $\int_a^b f(x)\,dx$ as a limit of Riemann sums
  - geometric interpretation of the definite integral of a nonnegative continuous function as area
  - geometric interpretation of the definite integral of an arbitrary continuous function as "signed area"
- Theorem on the integrability of a continuous function
- The Fundamental Theorem of Calculus, Part 1 (FTOC(1)): ie, the solution of the antidifferentiation problem
  - The function $g(x) = \int_a^x f(t)\,dt$
- The Fundamental Theorem of Calculus, Part 2 (FTOC(2)): ie, a computational technique for computing definite integrals
  - using definite integrals to compute areas
  - areas between curves
  - the substitution method for evaluating definite integrals
- indefinite integrals
- Properties of the integral (eg, Section 5.8, and equation 5.4.5 in the text)

Remember the following functions that frequently came up as examples / counterexamples:

$$y = |x|, \quad y = \frac{|x|}{x}, \quad y = \sin\frac{1}{x}, \quad y = x\sin\frac{1}{x}, \quad y = x^3 \text{ or } y = x^{1/3}$$

$$y = x^{-1}, \quad y = x^{-2}, \quad y = \pm x^4, \quad y = \sqrt{|x|}, \quad \text{the Dirichlet function}$$