

Test topics

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1 Selected solutions to Homework 5

Problem 1 5-3-1f

We solve each congruence in turn starting from the first:

1. $2x \equiv 1 \pmod{5} \implies \mathbf{x = 5t + 3}$.
2. Second congruence simplifies: $x \equiv 3 \pmod{2} \implies 5t \equiv 0 \pmod{2} \implies t \equiv 0 \pmod{2} \implies t = 2w, \mathbf{x = 10w + 3}$.
3. $4x \equiv 1 \pmod{7} \implies 40w \equiv -11 \pmod{7} \implies -2w \equiv -4 \pmod{7} \implies w \equiv 2 \pmod{7} \implies w = 7y + 2 \implies \mathbf{x = 70y + 23}$.
4. $5x \equiv 9 \pmod{11} \implies 350y \equiv -106 \pmod{11} \implies -2y \equiv 4 \pmod{11} \implies y \equiv -2 \pmod{11} \implies y = 11z + 9 \implies \mathbf{x = 653 + 770z}$

Problem 2 5-4-3

We need to solve the congruences $N \equiv -i + 1 \pmod{\Phi_i}$ for $i = 1, \dots, n$. Since the Φ_i are pairwise coprime (by 8-1-2) this is possible via the Chinese remainder theorem.

We have $\Phi_1 = 5, \Phi_2 = 17, \Phi_3 = 257$, we solve the three congruences starting with the *largest modulus*

1. $N \equiv -2 \pmod{257} \implies \mathbf{N = 257x - 2}$
2. $N \equiv -1 \pmod{17} \implies 257x \equiv 1 \pmod{17} \implies 2x \equiv 18 \pmod{17} \implies x \equiv 9 \pmod{17} \implies x = 17u + 9 \implies \mathbf{N = 4369u + 2311}$
3. $N \equiv 0 \pmod{5} \implies 4369u \equiv -2311 \pmod{5} \implies 4u \equiv 4 \pmod{5} \implies u = 5z + 1. \mathbf{N = 21845z + 6680}$

So **6680** is the smallest solution that works for Φ_1, Φ_2, Φ_3 .

The smallest solution for Φ_1, Φ_2 is **50**. (Remainder of **6680** mod $\Phi_1\Phi_2 = 85$).

Problem 3 5-4-5

We solve the congruences $N \equiv 0 \pmod{16 = 2^4}$, $N - 1 \equiv 0 \pmod{27 = 3^3}$, $N - 2 \equiv 0 \pmod{25 = 5^2}$ for $i = 1, \dots, n$.

1. $N \equiv 1 \pmod{27} \implies \mathbf{N = 27x + 1}$

2. $N \equiv 2 \pmod{25} \implies 27x \equiv 1 \pmod{25} \implies 2x \equiv 1 \pmod{25} \implies x = 25t + 13, \mathbf{N = 675t + 352}$

3. $N \equiv 0 \pmod{16} \implies 675t \equiv -352 \pmod{16} \implies 3t \equiv 0 \pmod{16}, t = 16w, \mathbf{N = 10800w + 352}$

The three consecutive integers are **350, 351, 352**.