$-$ denotes easy problems, $+$ is for hard ones

Lattice problems

(Lattice is the usual square lattice of points on the plane with integer coordinates)

L1$^-$ All strictly convex lattice pentagons have area at least 5/2.

L2 Large forests have some large dark spots.

A lattice point (x, y) is "visible" (from the origin) if $gcd\{x, y\} = 1$. Prove that for any r there is a lattice point $P = (a, b)$ such that no lattice point within distance r of P is visible.

L3$^+$ Lattice walks have many collinear points.

Starting from the origin we take a walk on the two-dimensional lattice by always jumping from a lattice point to one of its four neighbors. (A point may be visited more than once.)

Easy question: Is there necessarily a line on the plane which is visited infinitely many times?

Hard one: Prove that for every k there is a line on the plane which is visited at least k times.

Irrational numbers

I1$^-$ How many punches will destroy the whole plane?

Consider a paper punch that can be centered at any point of the plane and that, when operated, removes from the plane precisely those points whose distance from the center is irrational. How many punches are needed to remove every point?

I2 Gaps in $(n\alpha \ mod \ 1)$

Let α be irrational, and plot the points $a_k = k\alpha \ mod \ 1$, $k = 1, 2, \ldots, n$, on a circle of unit length. Show that at most three different distances (gaps) can occur between neighbors on the circle.

I3 A partition of natural numbers

Let α and β be positive irrationals such that $1/\alpha + 1/\beta = 1$. Define $a_n = \lfloor n\alpha \rfloor$ and $b_n = \lfloor n\beta \rfloor$. (Clearly, $a_1 < a_2 < \ldots$ and $b_1 < b_2 < \ldots$) Prove that $\{a_n\}$ and $\{b_n\}$ form a partition of $\mathbb{N}$, the set of all positive integers. (That is, every positive integer appears in one and only one of the two sequences.)

(And, by the way, L3 may belong to this group too!)