

1 Mouse in Cave

Problem

A mouse is trapped in a dark cave, with 3 doors. One of these doors lead outside through a 10-minute-long tunnel. The other two doors are connected through a 5-minute-long tunnel. The mouse chooses a door with equal probability ($1/3$) and runs down that tunnel. If he ends up back in the cave, he immediately chooses another door, again with equal probability ($1/3$). In particular, he forgets which door he just came out of, so he might choose that door again. What's the expected time before he escapes?

2 Random Date

Problem

Bob has managed to get a date with Alice at the Busch Campus Center. However, neither of them can remember the exact time they had agreed on. They both remember only that it was some time between 5PM and 7PM, so they decide to just pick a random time between 5PM and 7PM, go to the Campus Center, wait for half an hour (or until 7PM, whichever comes first), and if the other hasn't shown up, leave. What is the probability that Bob and Alice will catch each other?

3 The Truel

Problem

Three men, Mr. White, Mr. Grey and Mr. Black, have challenged each other to a truel (3-way version of a duel). However, they are of different skill levels with the pistol: Mr. White kills his target with probability $1/3$, Mr. Grey kills with probability $1/2$, and Mr. Black is a dead shot. So to make it fair, they decide to take turns shooting, starting with Mr. White, then Mr. Grey if he is still alive, then Mr. Black if he is still alive, then back to Mr. White, and so on, until only one man is left standing. What should Mr. White do?

4 Monty Hall's Dilemma

Problem

You are on Monty Hall's gameshow. There are 3 doors: behind 1 is a car, behind the other 2 are goats. You choose a door. Monty opens a different door, showing you a goat, and offers you the chance to switch your choice. Is it to your advantage to accept, decline, or does it not make a difference?

5 Extended Monty Hall's Dilemma

Problem

You're on another of Monty's gameshows, similar to the one above, but this time there are n doors. Again, behind 1 is a car, and behind the remaining $n - 1$ are goats. You choose a door. Then Monty repeats the following procedure $n - 2$ times: he opens a door other than the current choice, shows you a goat, and offers you the chance to switch. What should you do?

6 Sinbad and the Sultan

Problem

During one of his adventures, Sinbad saves the life of a powerful sultan. In gratitude, the sultan offers Sinbad a woman from his vast harem to take as a wife. However, there's a catch. The women will be presented to Sinbad one at a time, and he must make up his mind once and for all whether he chooses her. If he does, the procedure ends there. If he does not, he can never go back to her again. Moreover, Sinbad, who has seen many women in his travels and has very high standards, wants none but the most beautiful woman. What must be his strategy to maximize his chance of choosing her?