POLYNOMIAL EQUATIONS OVER 3 BY 3 MATRICES

Consider a polynomial equation

\[ X^n + A_{n-1}X^{n-1} + \ldots + A_0 = 0 \]

where \( A_0, \ldots, A_{n-1}, X \) are 3 by 3 matrices over the complex numbers. Such an equation may have infinitely many solutions (e.g., \( \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \) is a solution of \( X^2 = 0 \) for any complex number \( a \)). However, it is known that if the equation has a finite number, \( k \), of solutions then

\[ k \leq \binom{3n}{3}. \]

If \( k \) is any nonnegative integer in this range, is there a polynomial equation with exactly \( k \) solutions? It is known that the corresponding question for 2 by 2 matrices has an affirmative answer.