1. A region \( R \) in the first quadrant is bounded by the line \( y = x \) and the parabola \( y = -x^2 + 5x - 3 \).
   (a) Find the area of \( R \).
   (b) Find the volume of the solid obtained by rotating \( R \) around the \( y \)-axis.

2. Let \( R \) be the first quadrant region bounded by the curves \( y = e^x \), \( y = e^{-x} \) and the line \( x = 1 \).
   (a) What is the area of \( R \)?
   (b) What is the volume of the solid obtained by revolving \( R \) around the \( x \)-axis?
   (c) What is the volume of the solid obtained by revolving \( R \) around the \( y \)-axis?

3. Determine the value of each of the definite integrals. Express the answer in terms of mathematical constants such as \( \pi \) or \( e \), instead of numerical approximations.
   (a) \( \int_{\pi/8}^{\pi/4} \cos^4(x) \, dx \)
   (b) \( \int_0^1 \tan^{-1}(x) \, dx \)
   (c) \( \int_0^{\pi/2} \sin^3(x) \, dx \)

4. Calculate the following indefinite integrals:
   (a) \( \int e^{2x} \frac{dx}{1 + e^x} \)
   (b) \( \int \tan^2(x) \, dx \)
   (c) \( \int \frac{dx}{(-3 + 4x - x^2)^{3/2}} \)
   (d) \( \int x^3 \ln(x) \, dx \)
   (e) \( \int \frac{x^5}{x^4 - 1} \, dx \)
   (f) \( \int \frac{dx}{\sqrt{x^2 + 1}} \)

5. Solve the differential equation \( \frac{dy}{dx} = \cos^2(y) \) with the initial condition \( y(0) = \frac{\pi}{4} \). In the answer, express \( y \) explicitly as a function of \( x \).

6. Calculate each of the following improper integrals, or show that the integral is divergent:
   (a) \( \int_0^\infty \frac{\tan^{-1}x}{1 + x^2} \, dx \)
   (b) \( \int_0^\infty \frac{dx}{(x + 1)(x + 2)} \)
   (c) \( \int_0^1 \frac{x \, dx}{(1 - x^2)^{3/2}} \)

7. A metal rod of 300° F. cools to 200° F. in one minute when placed in a liquid of temperature 40° F.
   (a) What is the cooling constant?
   (b) At what time is the temperature 150° F.?

8. Suppose that \( a \) is a constant with \( a > 1 \). Determine each of the sequential limits:
   (a) \( \lim_{n \to \infty} \frac{n^2}{a^n} \)
   (b) \( \lim_{n \to \infty} (1 + \frac{a}{n})^n \)
   (c) \( \lim_{n \to \infty} \frac{\ln(a^n + n^2)}{n} \)
   (d) \( \lim_{n \to \infty} (\sqrt{n^2 + an} - n) \)

9. Test the following series for absolute convergence, conditional convergence, or divergence, explaining the test used and how it applies:
   (a) \( \sum_{n=1}^\infty \frac{n}{n^2 + 1} \)
   (b) \( \sum_{n=1}^\infty \frac{(-1)^n \ln n}{n \ln(n^2 + 1)} \)
   (c) \( \sum_{n=1}^\infty \frac{(n!)^3}{(3n)!} \)
   (d) \( \sum_{n=2}^\infty \frac{1}{n \sqrt{\ln n}} \)
   (e) \( \sum_{n=1}^\infty \frac{(-1)^n n^2}{2^n} \)
   (f) \( \sum_{n=1}^\infty \frac{(-1)^n n}{n^2 + 1} \)
10. Find the sum of the series \( \sum_{n=0}^{\infty} \frac{2^n + 5^n + 6^n}{8^n} \).

11. For which values of the positive constant \( a \) does the series \( \sum_{n=0}^{\infty} \frac{1}{a^n} \) converge?

12. Determine the radius and interval of convergence of the following power series. In addition, determine those points at which the series is absolutely convergent.
\[
\sum_{n=1}^{\infty} \frac{x^{3n}}{n^3 3^n}
\]

13. Find the Maclaurin series and the radius of convergence of the function \( f(x) = \frac{x}{x^4 + 16} \).

14. Using \( \ln(1 + x) = \int_0^x \frac{dt}{1 + t} \) and term-by-term integration, verify that the Maclaurin series of \( \ln(1 + x) \) is \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n + 1} \), if \( |x| < 1 \).

15. Let \( F(x) = \int_0^x \tan^{-1}(t^2) \, dt \).
   (a) Find the Maclaurin expansion of \( F(x) \).
   (b) Use this series to find a numerical series that converges to the integral \( \int_0^{1/2} \tan^{-1}(t^2) \, dt \).

16. Let \( f(x) = x^{1/4} \).
   (a) Find the third Taylor polynomial, \( T_3(x) \), of \( f(x) \) centered at \( a = 16 \).
   (b) Use Taylor’s inequality to give an upper bound for \( |x^{1/4} - T_3(x)| \) when \( 16 \leq x \leq 18 \).

17. The parametric curve
   \[
x(t) = \cos^3(t), \quad y(t) = \sin^3(t)
   \]
is often called the astroid.
   (a) Sketch the astroid.
   (b) Find \( \frac{dy}{dx} \) as a function of \( t \).
   (c) Determine the equation of the tangent line to the curve at \( t = \pi/6 \).
   (d) Find the arc length of the astroid for \( 0 \leq t \leq \pi/2 \), and the total arc length of the astroid.

18. Sketch the polar curve \( r = \theta^2 \) for \( 0 \leq \theta \leq \pi \), and find the area bounded by this curve and the \( x \)-axis.